Provably-Safe Autonomous Navigation of Traffic Circles

Rohit Konda, Eric Squires, Pietro Pierpaoli, Magnus Egerstedt, and Samuel Coogan

***Abstract*— As the decision making responsibilities of autonomous vehicles increase, they will be expected to navigate complex, unstructured environments such as traffic circles. These environments necessitate effective safety control algorithms. Control barrier functions provide such a tool for guaranteeing system safety by ensuring that control actions render a given safe set forward invariant. However, finding an appropriate control barrier function is challenging. To alleviate this challenge, we consider a nominal evasive maneuver for the system. Then a control barrier function is designed by considering the closed loop dynamics resulting from this hypothetical evasive maneuver. Using this approach in this paper, we propose a control algorithm to navigate an autonomous vehicle through a traffic circle in the presence of other vehicles. The synthesized control barrier function is able to simultaneously ensure lane keeping while avoiding collisions with other vehicles. The solution approach is then physically demonstrated on the Robotarium remote access testbed.**

I. INTRODUCTION

Autonomous vehicles are safety critical systems that have to operate in scenarios sensitive to the welfare of both pedestrians and passengers. To avoid the possibility of severe malfunctions, demonstrably safe control action is necessary for the design of such systems [1].

A rich literature exists studying the navigation of autonomous vehicles in certain restrictive environments such as lane-keeping and adaptive cruise control; see, e.g., [2]–[4]. Formal methods have been applied for safety verification in autonomous vehicles [5]–[7]. Variations of Model Predictive Control have also been an approach for safe autonomous navigation [8].

While the construction of controllers for these restrictive environments is maturing, the development of provably correct controllers to tackle more general environments, where correct driving behavior is not well-defined, is also necessary to achieve fully autonomous navigation. Examples of such less structured environments include parking lots, offroad environments, neighborhoods, and traffic circles.

This paper studies the problem of navigating a vehicle through a traffic circle, which includes the decision making of entering and exiting the traffic circle correctly, lane keeping that respects the curvature of the road, and collision avoidance behaviors. Literature of autonomous traffic circle navigation is sparse. Existing research includes [9] and [10], which implement a path planner to maintain lane keeping in a traffic circle, and [11], which implements a fuzzy logic controller to maintain lane keeping and collision avoidance. However, provable guarantees of behavior and robust collision avoidance behavior are not addressed in these works.

We propose developing provably correct control strategies for traffic circles using *control barrier functions* [12]. Control barrier functions allow for the generation of provably safe controllers for control affine systems through a computationally efficient quadratic program. A main advantage of the control barrier formulation is the decoupling of safety and performance conditions. Development of a controller that addresses many complex performance and safety specifications simultaneously is inefficient, and verification of these complex controllers may be computationally infeasible. By implementing safety conditions as pointwise constraints on the controller, control barrier functions allow for the separate handling of performance and safety specifications. Control barrier functions have been shown to be effective for collision avoidance and connectivity maintenance in mobile robots [13]–[15], and for bipedal walking [16], [17].

In addition, [18] presents control barrier functions as an excellent approach for provable guarantees for lane-keeping and adaptive cruise control in autonomous vehicles. However, the tools developed are not applicable to the unique geometries and collision avoidance behaviors present in traffic circles. Specifically, a nonholonomic system is used in the present paper to model collision avoidance behaviors in $\mathbb{R}^2$ rather than only in-lane collision avoidance. Additionally, the highly curved geometries of the traffic circle are considered in the present approach.

While control barrier functions are effective tools for generating safe controllers, developing control barrier functions for complex systems may be difficult, especially for systems with high relative degree or with actuator constraints [13], [19]. The paper [20] proposed a backstepping approach to handle construction of control barrier functions for non-holonomic systems with high relative degree but does not consider actuator constraints. In this paper, we propose using nominal evasive maneuvers for the autonomous vehicle to construct provably correct control barrier functions.

This paper is organized as follows. Section II introduces control barrier functions and relevant background. Section III formally introduces the Traffic Circle Problem. Section IV discusses the safety constraints of lane keeping and collision avoidance. Section V details experimental results. Section VI provides concluding remarks.

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II. BACKGROUND

A. Control Barrier Functions

In this section, we review how control barrier functions are employed as a means for control synthesis [12]. We consider a control affine system, which includes the unicycle dynamical model of a vehicle introduced subsequently, of the form

$$\dot{z} = f(z) + g(z)u \tag{1}$$

where $z \in D \subseteq \mathbb{R}^n$, $f : \mathbb{R}^n \to \mathbb{R}^n$ and $g : \mathbb{R}^n \to \mathbb{R}^{n\times m}$ are locally Lipschitz continuous functions and $u \in U \subseteq \mathbb{R}^m$.

Suppose a safe set for the system is denoted by X_s and is defined as the superlevel set of a continuously differentiable output function $\rho(z) : \mathbb{R}^n \to \mathbb{R}$ so that

$$X_s \triangleq \{z \in D : \rho(z) \geq 0\}. \tag{2}$$

We will also denote the unsafe set as $X_u \triangleq D \setminus X_s$. For example, $\rho(z)$ should be negative for a vehicle if its position is outside of a road lane.

The control affine system (1) is considered safe with respect to a given set $S \subseteq X_s$ if for a given initial condition $z(0) \in S$ if there exists a control input with $u(t) \in U$ such that for all $t \in [0, \infty)$, $z(t) \in X_s$. Control barrier functions provide a tractable way for verifying that a control system can be rendered safe by a controller, as formalized below.

For a given continuously differentiable output function $h(z) : \mathbb{R}^n \to \mathbb{R}$, define the corresponding superlevel set as $\mathcal{C}_h = \{z \in D : h(z) \geq 0\}$.

A continuous function $\kappa : (-b, a) \to \mathbb{R}$ for $b > 0, a > 0$ is considered an extended class $\mathcal{K}$ function if it is strictly increasing and $\kappa(0) = 0$.

Definition 1 (Zeroing Control Barrier Function (ZCBF), [12])**.** *A continuously differentiable function $h(z)$ is a* ***zeroing control barrier function*** *if there exists a locally Lipschitz extended class $\mathcal{K}$ function α such that*

$$\sup_{u\in U} [L_f h + L_g h u + \alpha(h(z))] \geq 0 \; \forall z \in D \tag{3}$$

where $L_g h$ and $L_f h$ denote the corresponding Lie derivatives, i.e, *$L_g h = \frac{\partial h(z)}{\partial z} g(z)$ and $L_f h = \frac{\partial h(z)}{\partial z} f(z)$.*

Theorem 1 (Corollary 1 in [12])**.** *If a state feedback control strategy $u(z)$ is employed for the system* (1) *that satisfies $u(z) \in K(z)$ for all $z \in D$, where*

$$K(z) = \{u \in U : L_f h + L_g h u + \alpha(h(z)) \geq 0\}, \tag{4}$$

then the set C_h is forward invariant, i.e., if $z(0) \in \mathcal{C}_h$ then $z(t) \in \mathcal{C}_h$ for all $t \in [0, \infty)$.

For control affine systems, notice that for fixed z, the constraint defining $K(z)$ in (4) is affine in u. Thus, given some nominal feedback controller $u_{\text{nom}}(z)$, it is possible to employ a quadratic program pointwise in the state in order to obtain a minimally invasive controller that guarantees safety according to [12]

$$u^*(z) = \underset{u\in K(z)}{\arg\min} \left\|u - u_{\text{nom}}\right\|^2 \tag{5}$$

B. Evasive Maneuvers For Actuation Constraints

When constructing a control barrier function for an autonomous system, care must be taken to ensure that $K(z)$ in (4) is nonempty for all safe states. Infeasibilities can occur if, for example, actuator constraints limit available control actions. One approach to prevent infeasibility, recently introduced in [19], is to assume a locally Lipschitz *nominal evasive maneuver* $\gamma : D \to U$ is available. Then, the execution of the controller γ results in the closed loop, autonomous dynamical system $\dot{z} = f(z) + g(z)\gamma(z)$.

Let $\Phi(t, z)$ denote the solution to this system when initialized at time 0 at state z. A suitable control barrier certificate can be obtained from this autonomous system in the following way, as proposed in [19]. Assume the safe set is characterized with the output function ρ as given in (2). Then let $h(z)$ for each $z \in D$ be the infimum value of ρ attained along solutions of the closed loop system assuming the nominal evasive maneuver, that is, let

$$h(z) = \inf_{\tau\in[0,\infty)} \rho(\Phi(\tau, z)). \tag{6}$$

It is established in [19] that, under mild conditions on ρ and on the dynamics of the system, h is indeed a valid ZCBF.

Although this construction provides a great theoretical tool, applicability is limited to systems that admit a closed form solution for the infimum. A finite time numerical approximation of the infimum can be used to calculate $h(z)$, but this loses the strict guarantees of feasibility of the barrier function. In this paper, we outline a similar approach of generating ZCBFs through verification of safety of the system under the evasive maneuver. However, by restricting the focus to unicycle models, geometric arguments can be utilized to generate closed form ZCBFs.

III. TRAFFIC CIRCLE PROBLEM

The geometry of the traffic circle is defined in accordance to the navigational objectives of the autonomous vehicle, as formalized in the definition below and shown in Figure 1.

Definition 2. *(Traffic Circle) A* traffic circle *is a triple (L, I, E) where $L \triangleq \{(x, y) \in \mathbb{R}^2 : \sqrt{x^2 + y^2} \in [r_{in}, r_{out}]\}$ denotes a ring with inner radius r_{in} and outer radius r_{out}, $I \subset \mathbb{R}^2$ is the initial entry region in a traffic circle with $I \cap L \neq \emptyset$, and $E \subset \mathbb{R}^2$ denotes the desired exit region for the traffic circle with $E \cap L \neq \emptyset$.*

We assume all vehicles in the system obey the unicycle dynamics given by

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{v} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} v\cos(\theta) \\ v\sin(\theta) \\ u_a \\ u_\omega \end{bmatrix} \tag{7}$$

where x, y are position coordinates of the vehicle, v is the forward speed of the vehicle, $\theta \in [0, 2\pi)$ is the heading of the vehicle, u_a is the input acceleration, and u_ω is the input angular velocity. The dynamics are control affine and can be written in the form (1) with state $z = (x, y, v, \theta)$ and

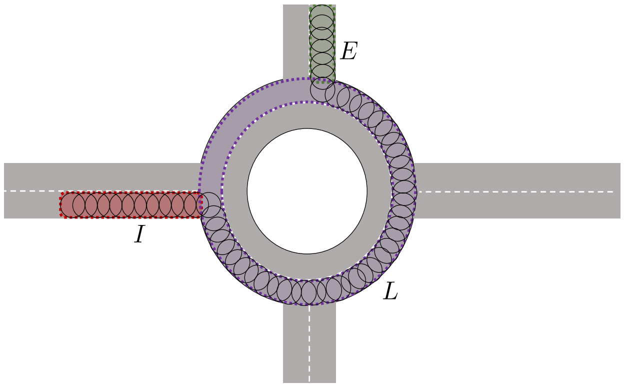

Fig. 1: An example of a traffic circle described in Definition 2, where an autonomous agent approaches from the west and exits to the north. Additionally, a cover of disks can be used for lane keeping, which is described in Section IV.

input $u = (u_a, u_\omega)$. We sometimes write $\dot{z} = F(z, u)$ for the unicycle dynamics (7).

We further assume input constraints of the form $u_a \in [-a_{max}, a_{max}]$ and $u_\omega \in [-\omega_{max}, \omega_{max}]$ for some maximum acceleration/deceleration a_{max} and some maximum angular velocity ω_{max}. These input constraints model the physical stopping and acceleration limits of the engine, as well as the angular limits on the wheels during turning, which are crucial in defining collision avoidance behavior of autonomous vehicles.

We are now in a position to define the main problem addressed in this paper.

Problem 1. *Given a traffic circle (L, I, E) and a primary vehicle A with position (x_A, y_A), design a control policy for vehicle A so that, if initialized in the entry region with $(x_A(0), y_A(0)) \in I$, the vehicle moves into the circle L, proceeds counterclockwise, and exits at region E,* i.e.*, $[x_A(t), y_A(t)]^T \in L \cup I \cup E\ \forall t \geq 0$ and $x_A(T), y_A(T) \in E$ for some finite time T. Moreover, ensure that the vehicle avoids collision with any other vehicles in the traffic circle so that $\left\|[x_A(t) - x_i(t), y_A(t) - y_i(t)]^T\right\| \geq D_s\ \forall t \geq 0$ for any other vehicle i.*

In other words, the agent must satisfy lane keeping and maintain the safety distance D_s from all other agents to be considered safe.

To solve the problem above, we construct a nominal controller that handles the performance of navigating a traffic circle. We then wrap this controller with the control barrier functions proposed in this paper to satisfy the safety specifications of lane-keeping and collision avoidance.

IV. SAFETY CONSTRAINTS

Geometry of traffic circles is highly varied with, *e.g.*, different numbers of traffic lanes, different curvatures, and different entry and exit points [21]. It would thus be undesirable to generate a handpicked control barrier function for each possible type of a traffic circle. Therefore in this paper, a control barrier function is instead generated for a simple geometric object, namely, disks, and is generalized to more complex sets through composition. Composition of barrier functions is a powerful way to handle complex safety specifications. For example, [15] constructs composed barrier functions through sums and products of piecewise differentiable barrier functions, and [22] achieves boolean composition through construction of nonsmooth barrier functions.

To this end, given a traffic circle (L, I, E) as in Definition 2, we generate an underapproximation of the set $L \cup I \cup E$ as the union of a collection of disks. In particular, let r^c be a fixed radius, and let (x_i^c, y_i^c) for $i = 1, \ldots, N$ be a collection of points so that

$$P \triangleq \bigcup_{i=1}^{N} \left\{(x, y) : \sqrt{(x - x_i^c)^2 + (y - y_i^c)^2} \leq r^c\right\} \subset L \cup I \cup E \tag{8}$$

and such that P provides a reasonable approximation to $L \cup I \cup E$; see Figure 1 for a demonstration of the construction of P.

This approach of describing lane geometry by a cover of disks was inspired by the results for lane tracking in [23]. Cover of disks are an efficient way to approximate various shapes, used in the graphics literature [24]. In this paper, we will not characterize exactly how to generate these covers given a tolerance. For intelligent constructions of covers of disks for arbitrary shapes, we refer the reader to [24]–[26].

Thus, we will now instead require that vehicle A remains within P while traversing the traffic circle, which implies the safety requirement that $(x_A, y_A) \in L \cup I \cup E$ of Problem 1. The requirement that $(x_A, y_A) \in P$ is then recast as follows. Let the safety output function be given by

$$\rho_i(x, y, v, \theta) = (r^c)^2 - (x - x_i^c)^2 - (y - y_i^c)^2. \tag{9}$$

Then the position of vehicle A is within the i-th disk with radius r^c and centerpoint (x_i^c, y_i^c) if and only if $\rho_i(x_A, y_A, v_A, \theta_A) \geq 0$ where $(x_A, y_A, v_A, \theta_A)$ is the state of vehicle A. Moreover, $(x_A, y_A) \in P$ if and only if $\max_{i=1,\ldots,N} \rho_i(x_A, y_A, v_A, \theta_A) \geq 0$.

Next, we study the problem of ensuring safety of each disk i using ZCBFs, and we will then compose these ZCBFs to ensure safety of the set P.

A. Turning Evasive Maneuver

For a particular ρ_i for some disk $i \in \{1, \ldots, N\}$, we first consider a turning evasive maneuver given by the feedback control strategy $(u_a, u_\omega) = \gamma^t(x, y, v, \theta) = (0, -\omega_{max})$ for all states (x, y, v, θ). An execution of this maneuver results in a circular trajectory of the vehicle, since the angular velocity is kept constant. Using a geometric argument, we define a candidate control barrier function, shown in Figure 2, as

$$h_i^t(x, y, v, \theta) = \left(r^c - \frac{v}{\omega_{max}}\right)^2 - \left(y - y_i^c - \frac{v\cos(\theta)}{\omega_{max}}\right)^2 - \left(x - x_i^c + \frac{v\sin(\theta)}{\omega_{max}}\right)^2. \tag{10}$$

Lemma 1. *$h_i^t(x, y, v, \theta)$ in* (10) *for the i^{th} disk comprising P in* (8) *is a valid ZCBF with respect to the safety function in* (9).

Proof. Let $z = (x, y, v, \theta)$ be the state of the vehicle. The dynamical system arising from the execution of $(u_a, u_\omega) = \gamma^t(z)$ is $\dot{z} = (v\cos(\theta), v\sin(\theta), 0, -\omega_{max})$. Note that $L_f h_i^t + L_g h_i^t \gamma^t = 0$. Let the domain be $D = \mathcal{C}_h$. Therefore for any extended class $\mathcal{K}$ function α, $L_f h_i^t + L_g h_i^t \gamma^t + \alpha(h_i^t) \geq 0$ on $h \geq 0$, so h_i^t is a ZCBF. It must also be shown that $C_{h_i^t}$ is indeed safe, i.e. that $h_i^t \leq 0$ whenever $\rho(z) \leq 0$. This can be shown by evaluating a constraint optimization program with the constraint $\rho(z) \leq 0$ and seeing that the maximum $h_i^t(z^*) = 0$. Therefore h_i^t is a valid ZCBF. ■

We can also use a similar ZCBF for encoding collision avoidance behaviors. First we construct a conjugated system between the autonomous agent A and another agent with index j. The resulting system for each pair is

$$\begin{bmatrix} \dot{z}_A \\ \dot{z}_j \end{bmatrix} = \begin{bmatrix} F(z_A, u_A) \\ F(z_j, u_j) \end{bmatrix} \tag{11}$$

where F is the unicycle dynamics as defined in Equation (7), $z_A = (x_A, y_A, v_A, \theta_A)$ is the state of agent A, $z_j = (x_j, y_j, v_j, \theta_j)$ is the state of agent j, $u_A = (u_{a,A}, u_{\omega,A})$ is the control input for agent A, and $u_j = (u_{a,j}, u_{\omega,j})$ is the control input for agent j. Then we encode a shared nominal evasive maneuver $(u_A, u_j) = \Gamma^T(z_A, z_j) = (\gamma^t(z_A), \gamma^t(z_j))$. This introduces the assumption that the other agent will execute the same evasive maneuver. However, this construction works well in experimental simulations provided that the other agents behave reasonably.

The collision avoidance safety function of the conjugated system is given by

$$\rho_{\text{coll}}(z_A, z_j) = (x_A - x_j)^2 + (y_A - y_j)^2 - (2D_s)^2 \tag{12}$$

where D_s describes a minimum safety distance between vehicles. Using a geometric argument, similar to (10), we construct the ZCBF

$$\begin{aligned} h^T(z_A,&z_j) = \\ &\left(\left(y_A - \frac{v_A \cos(\theta_A)}{\omega_{max}} \right) - \left(y_j - \frac{v_j \cos(\theta_j)}{\omega_{max}} \right) \right)^2 \\ &+ \left(\left(x_A + \frac{v_A \sin(\theta_A)}{\omega_{max}} \right) - \left(x_j + \frac{v_j \sin(\theta_j)}{\omega_{max}} \right) \right)^2 \\ &- \left(2D_s + \frac{v_A}{\omega_{max}} + \frac{v_j}{\omega_{max}} \right)^2. \end{aligned} \tag{13}$$

where ω_{max}^A and ω_{max}^j denote the angular velocity limits for vehicles A and j, respectively.

Lemma 2. *$h^T(z_A, z_j)$ in* (13) *for vehicle A and vehicle j in* (11) *is a valid ZCBF with respect to the safety function $\rho_{coll}(z_A, z_j)$ in* (12).

The proof is essentially identical to Lemma 1 and is omitted.

B. Braking Evasive Maneuver

In some situations in the roundabout, it is not possible to assume that the turning maneuver $\gamma^t(z)$ is available to the vehicle to execute in the lane. Therefore we also consider the evasive maneuver $\gamma^b(z) = (-\text{sign}(v) a_{max}, 0)$, assuming vehicle A has nonnegative velocity and $\gamma^b(z)$ is applied until $v = 0$. A similar approach for double integrator models was presented in [13]. A candidate control barrier function for each disk i can be constructed by examining distance of the midpoint of the trajectory to the edge of the disk, as illustrated in Figure 2, is

$$\begin{aligned} h_i^b(x, y, v, \theta) = &\left(r^c - \frac{v^2}{4a_{max}} \right)^2 - \left(y - y_i^c + \frac{v^2 \sin(\theta)}{4a_{max}} \right)^2 \\ &- \left(x - x_i^c + \frac{v^2 \cos(\theta)}{4a_{max}} \right)^2. \end{aligned} \tag{14}$$

Lemma 3. *$h_i^b(x, y, v, \theta)$ in* (14) *for the i^{th} disk comprising P in* (8) *is a valid control barrier function with respect to the safety function in* (9).

Proof. Under the braking maneuver $\gamma^b(z)$, the solution $\Phi(t, z)$ is always within a ball with radius $\frac{v^2}{4a_{max}}$ and center $(x + \frac{v^2 \cos(\theta)}{4a_{max}}, y + \frac{v^2 \sin(\theta)}{4a_{max}})$ for $\forall t \geq 0$. If $h_i^b(x, y, v, \theta)$ is positive, then this ball is contained in $\{z : \rho_i(z) \geq 0\}$ for the safety function $\rho_i(z)$. Therefore if the system is initialized in $C_{h_i^b}$, the system is safe for all time under the evasive maneuver and h_i^b is a valid control barrier function. ■

Note that unlike h_i^t generated from the turning evasive maneuver, h_i^b is not shown to be a ZCBF. However the control barrier function h_i^b can still be used in a QP Program. If the QP ever becomes infeasible, the vehicle can always switch to its evasive maneuver to maintain safety, similar to (17) in [13].

Braking behavior can also be used for collision avoidance. For the pair of vehicles A and j, an evasive maneuver $\Gamma^B(u_A, u_j) = (\gamma^b(z_A), \gamma^b(z_j))$ is used. Similar to (14), the following control barrier function can be constructed:

$$\begin{aligned} h^B(z_A,&z_j) = \\ &\left(\left(y_A + \frac{v_A^2 \sin(\theta_A)}{4a_{max}} \right) - \left(y_j + \frac{v_j^2 \sin(\theta_j)}{4a_{max}} \right) \right)^2 \\ &+ \left(\left(x_A + \frac{v_A^2 \cos(\theta_A)}{4a_{max}} \right) - \left(x_j + \frac{v_j^2 \cos(\theta_j)}{4a_{max}} \right) \right)^2 \\ &- \left(2D_s + \frac{v_A^2}{4a_{max}} + \frac{v_j^2}{4a_{max}} \right)^2 \end{aligned} \tag{15}$$

Lemma 4. *$h^B(z_A, z_j)$ in* (15) *for vehicle A and vehicle j in* (11) *is a valid control barrier function with respect to safety function $\rho_{coll}(z_A, z_j)$ in* (12).

The proof is essentially identical to Lemma 3 and is omitted.

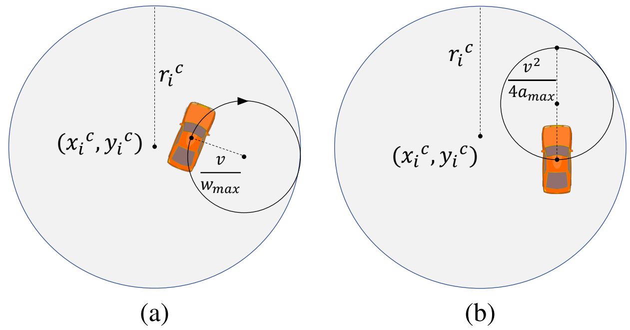

Fig. 2: (a) An autonomous vehicle executing an evasive maneuver of $[0, -\omega_{max}]$. In this figure, $h_i^t(x, y, v, \theta) = 0$. (b) An autonomous vehicle executing an evasive maneuver of $[-a_{max}, 0]$. In this figure, $h_i^b(x, y, v, \theta) = 0$ as well.

C. Composition

Encoding both evasive maneuvers γ^t and γ^b for collision avoidance into one control barrier function is done through composition. Furthermore a total control barrier function for the lane of the traffic circle is defined by composing the control barrier functions for the disks comprising P.

A total control barrier function for avoiding collision between vehicle A and vehicle j is defined as

$$h(z_A, z_j) = \max\{h^B(z_A, z_j), h^T(z_A, z_j)\}. \quad (16)$$

For lane keeping, h_i^t and h_i^b for the corresponding i-th disk are composed to give a total control barrier function h_{lane} as

$$h_{\text{lane}}(x, y, v, \theta) = \max\{h^t(x, y, v, \theta), h^b(x, y, v, \theta)\}. \quad (17)$$

where $h^t(x, y, v, \theta) = \max_{i=1,\ldots,N} h_i^t(x, y, v, \theta)$, and $h^b(x, y, v, \theta) = \max_{i=1,\ldots,N} h_i^b(x, y, v, \theta)$. The max operator $\max\{h_1, h_2\}$ of 2 ZCBFS gives a corresponding superlevel set $\mathcal{C}_{\max\{h_1,h_2\}} = C_{h1} \cup C_{h2}$. If C_{h1} and C_{h2} are shown to be control invariant by their respective ZCBFs, then $C_{h1} \cup C_{h2}$ is necessarily control invariant. Composition with the max operator introduces points of nondifferentiability into the barrier function, which has been studied in [22]. Alternatively, to avoid using nonsmooth analysis, the ZCBF h_{lane} in (17) and $h(z_A, z_j)$ in (16) can be smoothly underapproximated. In particular, consider the α-softmax function [27] applied to h_{lane} resulting in

$$\hat{h}(x, y, v, \theta) = \frac{\sum_i^n h_i(x, y, v, \theta) e^{a h_i(x,y,v,\theta)}}{\sum_i^n e^{a h_i(x,y,v,\theta)}}. \quad (18)$$

This construction smoothly under-approximates the maximum so that $\hat{h}(x, y, v, \theta) \leq \max_{i=1,\ldots,N} h_i(x, y, v, \theta)$ for all x, y, v, θ and therefore $\mathcal{C}_{\hat{h}_{\text{lane}}} \subseteq \mathcal{C}_{h_{\text{lane}}}$. Here, $a \geq 0$ is a design parameter determining closeness of fit. For large enough a, it is possible to get arbitrary close under approximations of h_{lane} while retaining differentiability of the control barrier function. Therefore we can use $\hat{h}_{\text{lane}}$ as a substitute to evaluate for a controller in a quadratic program. The QP Program for $\hat{h}_{\text{lane}}$ may not be strictly feasible, but because $\hat{h}_{\text{lane}} \leq h_{\text{lane}}$ and $\mathcal{C}_{\hat{h}_{\text{lane}}} \subseteq \mathcal{C}_{h_{\text{lane}}}$, an evasive maneuver is always available to keep the autonomous agent safe in $C_{h_{\text{lane}}}$.

V. RESULTS AND SIMULATIONS

Experimental results were collected on the Robotarium, a remote-access and multi-agent testbed [28]. In this demonstration, as shown in Figure 3[1], an autonomous vehicle navigates a traffic circle in the presence of two other vehicles. Safety constraints are encoded through the control barrier functions outlined in this paper. To handle performance constraints, a simple proportional controller is used:

$$\begin{bmatrix} u_{a,\text{nom}} \\ u_{\omega,\text{nom}} \end{bmatrix} = \begin{bmatrix} K_1(v_{\text{desired}} - v_A) \\ K_2(\sin(\theta_{\text{desired}} - \theta_A)) \end{bmatrix} \quad (19)$$

where v_{desired} is a constant desired velocity, and K_1 and K_2 are gains of the controller. $\theta_{\text{desired}} = \text{angle}(\frac{k_1}{|\phi - \phi_{in}|}\text{vec}(\phi + \pi) + \frac{k_2}{|\phi - \phi_{out}|}\text{vec}(\phi) + k_3\text{vec}(\phi + \frac{r}{p_{\text{lane}}}\frac{\pi}{2})$, where k_1, k_2, k_3 are relative gains for each term, (r, ϕ) is the position of the vehicle in polar coordinates, $\text{vec}(\phi) = [\cos(\phi), \sin(\phi)]$, $\text{angle}([x, y]) = \arctan(y/x)$, and ϕ_{in} and ϕ_{out} are the directions of the entry and exit regions with respect to the origin. This defines a desired vector field that points into the traffic circle in the initial region, points out of the traffic circle in the exit region, and converges to the middle of the lane (p_{lane}) while in the traffic circle. Values of $K_1 = 0.1$, $K_2 = 5$, $v_{\text{desired}} = 0.1$ m/s, $k_1 = k_2 = 0.2$, $k_3 = 2$, $\phi_{in} = -\pi$, $\phi_{out} = \frac{\pi}{2}$, and $p_{\text{lane}} = 0.7$ m were used. In the experimental scenario shown in Figure 3, two vehicles, representing human drivers who have the right of way, are already present in the traffic circle. They are also modeled with the proportional controller in (19), but are not equipped with any safety modules. However, the autonomous vehicle A is still able to safely navigate the traffic circle.

VI. CONCLUSIONS

In this work we examine how to navigate an autonomous vehicle safely through a traffic circle. As autonomous vehicles are safety-critical systems, safety specifications are an integral part of the design of the controller. We introduce control barrier formulations as a way to handle safety constraints, specifically collision avoidance and lane keeping in a roundabout. We construct control barrier functions for simple safety specifications through geometric arguments. Composition is then used to extend the control barrier functions to handle the traffic circle problem. Experimental results are shown to validate the approach using the Robotarium testbed.

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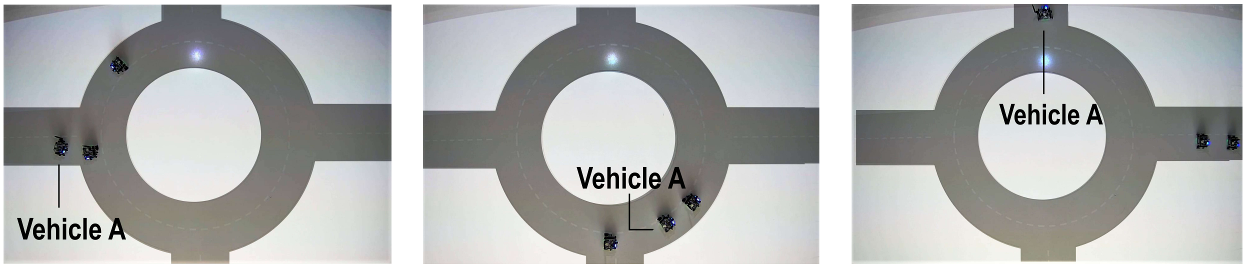
[1] A video of the experimental results are available at `https://youtu.be/_nugBe6nZaE` at 10x speed. All source code is available at `https://github.com/gtfactslab/CCTA19_PSANTC`



(a) (b) (c)

Fig. 3: Experiments on the Robotarium. (a) The autonomous agent A enters the traffic circle from the left (marked with Vehicle A). Since the collision avoidance barrier is active, agent A slows down and waits for the first vehicle to pass, then merges into the roundabout. (b) All vehicles traverse the roundabout, with agent A maintaining strict lane keeping. Even though the preceding vehicle is traveling slower than the desired velocity of vehicle A, vehicle A is still able to maintain a safe distance. (c) After the preceding vehicle exits, vehicle A speeds up and exits the traffic circle at the top.

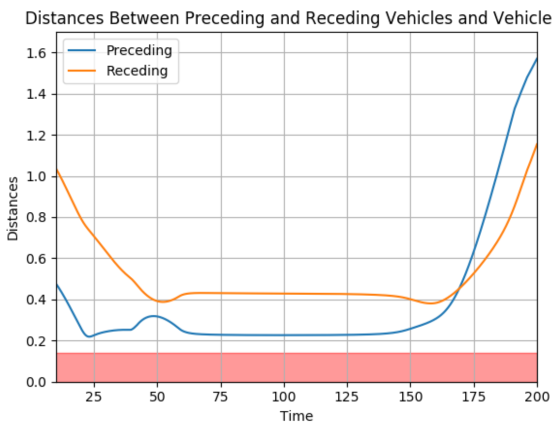


Fig. 4: Distances between vehicle A and the preceding and receding vehicles are depicted. The distances between the vehicles never violate the collision avoidance safety specification, shown as the shaded region.

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