A Compartmental Dynamical Network Flow Model for Evacuation Planning of Cities

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Abstract— **An evacuation due to, e.g., hurricanes, floods, or forest fires often puts severe stress on the transportation network and can create congestion or grid locks. In this paper, we present an evacuation planning solution where we model the city traffic as a compartmental model. Each compartment corresponds to one Traffic Analysis Zone (TAZ). Under normal (i.e., non-evacuation) operations, there is often a heterogeneity in the transportation network, which enables the traffic dynamics to be modelled through Macroscopic Fundamental Diagrams (MFDs). In the evacuation case, where all the traffic travels in the same direction, the typical shape of the MFD is not present. Because of this, we propose a dynamic system with a differently shaped relationship between the traffic flow and the amount of traffic present. We then utilize the model to propose two different evacuation planning strategies, a staggered release approach and an approximate model predictive control (MPC) approach. Micro-simulator studies performed for the city of Greensboro, NC, show that by using this evacuation planning methodology the total travel time needed to evacuate the city is reduced to around half without delaying the overall time it takes to evacuate.**

I. INTRODUCTION

When a severe disaster is imminent, such as a hurricane, flood, or forest fire, it is common practice to call for an evacuation of the at-risk region or city. If all the residents in the city are asked to leave simultaneously, the traffic demand can be higher than what the transportation network in the city can handle, causing grid locks or unnecessary delays. By providing an evacuation schedule for neighborhoods or smaller geographic sectors within the region, e.g., through text messages, smoother and more efficient evacuations are possible.

In this paper we develop a methodology for evacuation planning that takes into account the congestion effects within the city. We model the city traffic dynamics through a compartmental dynamical model. Each compartment corresponds to a sector of the city; in the case study, each sector is a Traffic Analysis Zone (TAZ) within the city, a geographic unit common in transportation planning in the United States usually consisting of a few thousand people. In the proposed model, the throughput through each compartment is determined by the number of vehicles present in the compartment. Since the throughput will decrease with the congestion level, it can be beneficial to not let all vehicles in the city depart at the same time, but rather tell some drivers to depart at a later time to avoid getting trapped in congestion. The controller's task is to determine the departure time of each vehicle to guarantee the most efficient overall evacuation.

Our modeling approach is similar to the ones utilizing Macroscopic Fundamental Diagrams (MFDs) [1], [2] to model city traffic, in the sense that we divide the city into compartments and model the traffic as a fluid continuous quantity. MFDs have previously been utilized for macroscopic-level control of the flow between different regions to improve the traffic flows [3], as well as routing of vehicles within a city [4]. However, due to the homogeneity of the traffic pattern in evacuation scenarios where everyone is traveling in the same direction, MFDs do not provide an appropriate model in this case. This since the MFD models include congestion effects within a region itself, something that will be overshadowed by the supply constraints of a downstream region when the traffic is heading towards one direction. As will be shown later in the paper, both by stress testing the traffic dynamics in a micro-simulator and comparing the simulated trajectories with trajectories from the micro-simulator, the traffic dynamics during evacuations within the city is seen to be closer to a queuing model with limited supply.

Although the city traffic dynamics is different from the highway traffic dynamics, there are similarities with the ramp metering problem for highway traffic [5], [6], [7]. In both problems, there are buffers with vehicles (on the on-ramps and parked vehicles, respectively), and the flow on the highway is usually modeled through the cell transmission model [8], [9], which is a non-linear compartmental flow model.

The evacuation planning problem has recently been studied in [10], [11]. In those papers, the focus is on releasing the vehicles such that the traffic flows stay below the capacities on the highways. No consideration is given to the traffic dynamics within the city, which is the focus of this paper.

The contributions of this paper are as follows: We develop a compartmental model for traffic flows during an evacuation scenario. We then utilize a high fidelity microscopic traffic simulator to calibrate our model and we show that our model successfully captures the congestion effects within the network and closely matches results from a detailed microscopic simulation. Once the model is calibrated, we illustrate how the model can be utilized to develop different evacuation plans. The performance of the evacuation plans are then evaluated in the microscopic traffic simulator.

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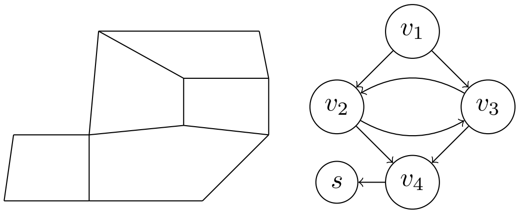

Fig. 1. Example of how different sectors in a region targeted for evacuation can be mapped to the corresponding graph representation.

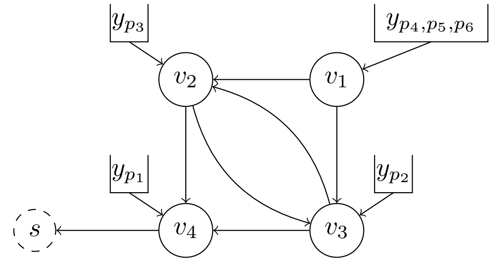

Fig. 2. Example of a city with four sectors, where each of the sector has one or several path dependent input buffers with vehicles that should be released. Once the vehicles are released, they will travel to the safe sector, node s. In this example, the set of paths is $\mathcal{P} = \{p_1 = (v_4, s), p_2 = (v_3, v_4, s), p_3 = (v_2, v_4, s), p_4 = (v_1, v_2, v_4, s), p_5 = (v_1, v_3, v_4, s), p_6 = (v_1, v_2, v_3, v_2, v_4, s)\}$.

The paper is outlined as follows: The rest of this section is devoted to introducing some basic notation. In Section II we present the compartmental model used for modeling the city traffic. In Section III we present our different strategies for generating evacuation plans based on the model, and in Section IV the model is calibrated and the evacuation plans are evaluated in a microscopic traffic simulator. The paper is concluded with a few pointers to future research.

A. Notation

We let $\mathbb{R}$ denote the set of real numbers, and $\mathbb{R}_+$ the set of non-negative reals. For a finite set $\mathcal{A}$, we let $\mathbb{R}^{\mathcal{A}}_{(+)}$ denote the set of (non-negative) vectors indexed by the elements of $\mathcal{A}$.

II. Model

The city is divided into sectors, where each sector is represented by a node in a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ is the set of nodes and $\mathcal{E}$ the set of links. A directed link between two sectors indicates that vehicles can travel from one sector (the tail of the link) to the other (the head) without passing through any other sector. An example of the mapping between the regions and the corresponding directed graph is shown in Fig. 1. Throughout the paper, we assume that all vehicles are traveling towards a common safe sector, and we will refer its corresponding node as the *safe node*, s. All other nodes, i.e., the ones that should be evacuated, will be referred to as *regular nodes*, and we will denote the set of those nodes $\mathcal{R}$. Hence, $\mathcal{V} = \mathcal{R} \cup \{s\}$.

Since not all the vehicles departing from one sector will necessarily take the same path, we introduce the set of paths $\mathcal{P}$. Each path $p \in \mathcal{P}$ consists of a tuple of nodes, i.e., $p = (\gamma_1, \gamma_2, \ldots \gamma_{\ell_p})$, where ℓ_p denotes the length of path p and $(\gamma_i, \gamma_{i+1}) \in \mathcal{E}$ for all $i \in \{1, \ldots, \ell_p - 1\}$. We will allow for a path to visit the same node several times, but not without visiting another node in between, so $\gamma_i \neq \gamma_{i+1}$ for all $i \in \{1, \ldots, \ell_p - 1\}$. Since all paths are assumed to end in the same safe sector, $\gamma_{\ell_p} = s$ for all paths.

As state space, we will use the traffic volume along each path in each sector together with the volume of parked vehicles waiting to be evacuated in each sector. We let $x^p \in \mathbb{R}^{\ell_p}_+$ denote the traffic volume in each sector along a given path $p \in \mathcal{P}$, and we let x^p_i denote the traffic volume at the ith node flowing along path p. Traffic along different paths can travel through the same sector, so with a slight abuse of notation, we let x_v denote the aggregate volume of vehicles in one sector $v \in \mathcal{V}$, i.e.,

$$x_v = \sum_{p \in \mathcal{P}} \sum_{1 \leq i \leq \ell_p | \gamma_i = v} x^p_i \,. \quad (1)$$

For each path, we also assign a buffer of parked vehicles that should be evacuated. Those vehicles have not entered the transportation network yet and hence they have not affected the traffic dynamics. We denote those buffers $y \in \mathbb{R}^{\mathcal{P}}_+$, where y_p is the volume of vehicles that will follow path $p = (\gamma_1, \gamma_2, \ldots, \gamma_{\ell_p}) \in \mathcal{P}$ once they depart and start their journey along in sector γ_1.

We let $z^p_{i,j}$ denote the intermediate flow between the ith sector and jth sector along path p. Moreover, we assume the departure rate of parked vehicles can be controlled, e.g., through sector based evacuation orders, where it is possible to tell a fraction of all the vehicles parked in a given sector that will follow a given path to depart at a given time. We denote this control action u_p where $p \in \mathcal{P}$ is the path the vehicles will follow. We note that controlling evacuations in this way is likely practically viable, as has been discussed in, e.g., [10], [11].

Example 1: Consider the network in Fig. 2, which is the same network as in Fig. 1 but with the buffers included. In this case, the network has four regular sectors, corresponding to the nodes v_1, v_2, v_3, v_4 and one safe sector, node $s = v_5$. In this example, the vehicles departing from sector v_1 can take several different paths. While there are only two paths that visit each sector once, (v_1, v_2, v_4, v_5) and (v_1, v_3, v_4, v_5), there might be a possibility that some of the vehicles' paths in the actual traffic network cross the sector boundary several times. If that is the case, paths such as $(v_1, v_2, v_3, v_2, v_4, v_5)$ might be possible as well.

To model the flow dynamics, we discretize time in steps of length $h > 0$. Let $k \in \{0, 1, ..., K\}$ be the discretization points for an evacuation over the horizon Kh. The dynamics for each path $p = (\gamma_1, \gamma_2, \ldots, \gamma_{\ell_p}) \in \mathcal{P}$ satisfy

$$x^p_1(k+1) = x^p_1(k) + u_p(k) - z^p_{1,2} \,, \quad (2)$$

$$x^p_i(k+1) = x^p_i(k) + z^p_{i-1,i} - z^p_{i,i+1} \,, \quad \forall i \in \{2, \ldots, \ell_p - 1\} \,, \quad (3)$$

$$y_p(k+1) = y_p(k) - u_p(k) \,, \quad (4)$$

where $z^p_{i,i+1}$ denotes the flow between two neighboring sectors along path p and $u_p(k)$ denotes the control action of departing vehicles. The control action $u_p(k)$ has always to be non-negative and chosen such that y_p always remains non-negative. For the model to be valid, the discretization step h has to be chosen small enough such that no vehicle will have time to cross more than one border between the zones.

The flow between two adjacent sectors is limited by the *supply* of the downstream sector. Since the capacity of the roads into a sector from a neighboring sector affects the supply, a supply function has to be assigned to each link in the graph. For a given link, the supply function is dependent on the number of vehicles within the head node of the link, i.e., the sector into which vehicles travel, such that when the sector contains enough vehicles that the roads become full, the supply is zero. We describe the supply functions by two parameters: the maximum inflow capacity from sector $v \in \mathcal{V}$ to sector $w \in \mathcal{V}$, denoted $c_{v,w}$, and the maximum storage capacity, denoted $\bar{c}_{v,w}$. We let the storage capacity depend on the neighboring sector as well, since a sector may be able to accept more traffic from one neighboring sector compared to another. Observations from the microscopic traffic simulator, which have been concluded through testing the evacuation scenario with different scaling of the demand, suggest that the supply function can be expressed as

$$s_{v,w}(x_w) = \begin{cases} c_{v,w} & \text{if } x_w \leq \bar{c}_{v,w}\,, \\ 0 & \text{otherwise.} \end{cases} \tag{5}$$

With the supply functions, we are able to complete our model. The flows between the sectors is either limited by the number of vehicles present in the upstream sector or the supply function, and hence given by

$$z^p_{i,j}(x) = \min\left(x^p_i, \frac{x^p_i}{x_{\gamma_i}} \cdot h \cdot s_{\gamma_i,\gamma_j}(x_{\gamma_j}) \right)\,. \tag{6}$$

Recall that x_{γ_i} is the aggregate traffic volume for the ith node along path p, and given by (1). This completes the description of the dynamics. In the next section we will discuss how to control the vehicles' departure time, i.e., u, by utilizing the proposed model.

III. Evacuation Planning Strategies

In this section we describe the different evacuation planning strategies that will later be evaluated.

A. Static Departure Rate Strategies

To both evaluate the correctness of our dynamical model (2)–(6) and to establish base cases for comparison with more advanced evacuation strategies, we consider several static naive strategies. The first naive strategy is to release all vehicles at once. We also consider naive strategies that release a fixed number of vehicles at each discretization step for every path, as long as there are vehicles waiting to be released.

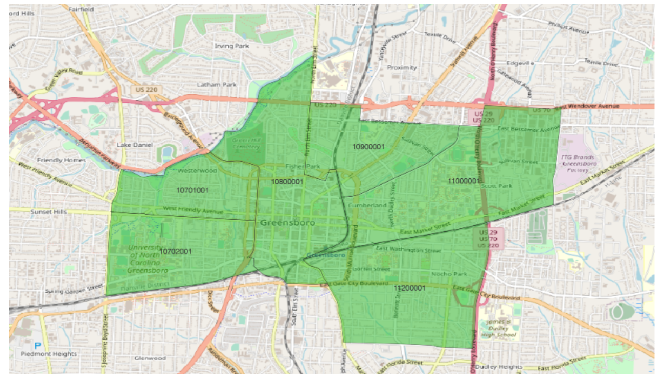

Fig. 3. The area of the central part of Greensboro, NC, to be evacuated. The selected area contains six Traffic Analysis Zones (TAZ).

B. Staggered Release Strategy

By utilizing the network flow model (2)–(6), it is possible to develop staggered release strategies, where the release of vehicles into one sector starts when the vehicle buffers of the downstream sectors are below a certain threshold. This strategy is a naive approach to avoid an upstream release of vehicles that creates too much congestion downstream.

C. Approximate MPC Strategy

To compute an optimal evacuation plan, we formulate the control problem as an MPC problem. Since the original problem is non-linear due to the supply functions in (5), to simplify computation, we relax the dynamics constraints such that the flow between the sectors is instead

$$z^p_{i,j}(x) = \min\left(x^p_i, h \cdot s_{\gamma_i,\gamma_j}(x_{\gamma_j}) \right)\,.$$

While this relaxation allows for more flow between sectors when several paths use the same link, we add an additional capacity constraint on the amount of vehicles allowed in each sector in part to compensate for this. We will denote this maximum storage capacity $\bar{x}_v$ for a sector $v \in \mathcal{R}$.

The objective is to minimize the total travel time, where the time waiting to depart counts as travel time as well. Hence the approximate MPC problem reads:

$$\begin{aligned} &\text{minimize}_{u(k)\in\mathbb{R}^{\mathcal{P}}_+} \sum_{k=1}^{K} \left(\sum_{v\in\mathcal{R}} x_v(k) + \sum_{p\in\mathcal{P}} y_p(k) \right) \\ &\quad \text{subject to } (2), (3), (4), (5), \\ &\qquad\qquad z^p_{i,j}(x) = \min\left(x^p_i, \cdot h \cdot s_{\gamma_i,\gamma_j}(x_{\gamma_j}) \right)\,, \\ &\qquad\qquad x_v(k) \leq \bar{x}_v\,, \quad \forall v \in \mathcal{R}\,. \end{aligned}$$

IV. Validation in Micro Simulator

A. Scenario Setup

To validate the proposed solution, we constructed a SUMO [12] scenario for part of the city of Greensboro, NC, show in Fig. 3. The area of interest contains 6 TAZs. The road network was imported from OpenStreetMap. To generate the traffic demands, we used open parcel data

TABLE I

THE NUMBER OF VEHICLES IN EACH TAZ

Short ID	TAZ	Vehicles
A	10701001	1129
B	10702001	1149
C	10800001	938
D	10900001	673
E	11000001	485
F	11200001	1459

TABLE II

NUMBER OF VEHICLES FOR EACH PATH

No.	Path	Vehicles	No.	Path	Vehicles
1	B	1149	9	E, D, C, A	15
2	A	1129	10	D, C, A	602
3	C	240	11	C, F, C	8
4	C, A	578	12	C, D, C, A	18
5	C, B	37	13	C, E, C, A	1
6	E, F, B	470	14	D, C	32
7	F, C	1266	15	C, D, C, D, C, A	56
8	F, E, C, A	193	16	D, C, D, C, A	39

from the state of North Carolina[1] and mapped each parcel to its corresponding edge in the SUMO network. Then, depending on the parcel category, we obtain an estimate of the number vehicles at each parcel. For parcels where the number of vehicles could vary significantly, e.g., apartment complexes, we manually estimated the number of vehicles through Google Maps, where both the size of the buildings and the number of visible parking spots were used to provide an estimate. The number of vehicles in each TAZ is shown in Table I.

For the path generation in SUMO, we assumed that all the vehicles wanted to evacuate towards the west and imposed that all vehicles should be on the highway I40 towards Winston-Salem, NC, once they have left the city. Since in this paper, we assume that we cannot control the routing of the vehicles, each vehicle is assumed to take its shortest path (with respect to free flow travel time). The paths were generated through the SUMO tool `duarouter`[2]. Since the focus of this work is to model the city traffic dynamics of the evacuation process, the vehicles will disappear from the simulation once they exit the modeled region.

The generated paths, grouped by the TAZs they are passing through, are summarized in Table II.

B. Parameter Identification

To identify the parameters for the supply functions, we generated several versions of the evacuation scenario and ran all the versions with SUMO. We scaled up the demands with a factor of 2, 4, and 8, and also spread the vehicles' departure rates uniformly over time windows of 1, 2, and 4 hours. By then plotting the flow between sectors for those different combinations of demand and time frame, we were able to identify both the maximum flow possible between the different sectors, as well as the maximum storage capacity. The latter was identified by observing when vehicles start to flow again, after the number of vehicles in the downstream sector has decreased below a certain threshold.

[1]https://www.nconemap.gov/pages/parcels

[2]https://sumo.dlr.de/docs/duarouter.html

TABLE III

SUPPLY FUNCTION PARAMETERS

From TAZ	To TAZ	Max Flow [vehicles/hour]	Max Capacity [vehicles]
10800001 (C)	10701001 (A)	3000	700
10800001 (C)	10702001 (B)	2500	300
10800001 (C)	10900001 (D)	1750	400
10800001 (C)	11000001 (E)	700	1000*
10800001 (C)	11200001 (F)	400	1000*
10900001 (D)	10800001 (C)	5000	400
11000001 (E)	10800001 (C)	2500	700
11000001 (E)	10900001 (D)	2500*	1000*
11000001 (E)	11200001 (F)	2500*	1000*
11200001 (F)	10702001 (B)	2500*	1000*
11200001 (F)	10800001 (C)	2500	900
11200001 (F)	11000001 (E)	1000	800

* Too few vehicles were detected during the identification experiments, so the numbers are set to a standard value.

The identified parameter values for the supply functions are summarized in Table III. For a few cases, we observed vehicle flow between the sectors that was too small to provide meaningful values, and in those cases we used a nominal estimate instead.

C. Controller Implementation

For all the controllers that use the dynamical network flow model (2)–(6) we assumed a discretization step of $h = 5$ minutes.

1) Static Departure Rate: In this case study we consider three different strategies that release the vehicles by a fixed departure rate. One that releases all the vehicles at once, and two others that release 200 or 50 vehicles along each path every 5 minutes.

2) Staggered Release: In the staggered release approach, we release 300 vehicles every 5th minute in TAZs A and B. Once the total number of vehicles waiting to be released in those two TAZs is less than 300, we start to release vehicles departing from TAZ C, also with the rate of 300 vehicles each 5th minute. And once the number of vehicles waiting to be released into TAZ C is less than 300, the remaining vehicles starting their journeys in TAZs D, E, or F are released.

3) Approximate MPC: In the implementation of the approximate MPC, a value of $\bar{x}_v = 300$ was used for all sectors $v \in \mathcal{R}$. Moreover, for computational reasons we only considered paths with 50 or more vehicles. For paths with a smaller number of vehicles, all the vehicles were released immediately. To numerically solve the MPC problem, YALMIP [13] together with GUROBI [14] was used.

D. Simulation Results

The results of SUMO simulations of the different strategies are summarized in Table IV. The table shows the average values for each trip, together with the time that the last vehicle leaves the region and the total travel time. The "Average duration" is the time it takes for the vehicle to exit the region after it has been inserted on a road. The "Average waiting time" is the time the vehicle spends involuntary

TABLE IV

STATISTICS FROM THE SUMO TRAFFIC SIMULATOR FOR THE DIFFERENT EVACUATION PLANS

	All at once	Fixed rate 200	Fixed rate 50	Staggered release	Approximate MPC
Average speed [m/s]	3.24	4.70	9.73	5.12	5.38
Average duration [s]	1473.07	1101.93	305.42	856.20	742.26
Average waiting time [s]	1064.75	743.98	104.02	556.54	436.47
Average time loss [s]	1370.40	999.49	202.94	753.50	639.67
Average departure delay [s]	226.89	122.18	4.92	81.89	71.30
Time to evacuate [s]	4600	4450	7750	5300	4200
Total travel time [h]	2754	1983	503	1519	1318

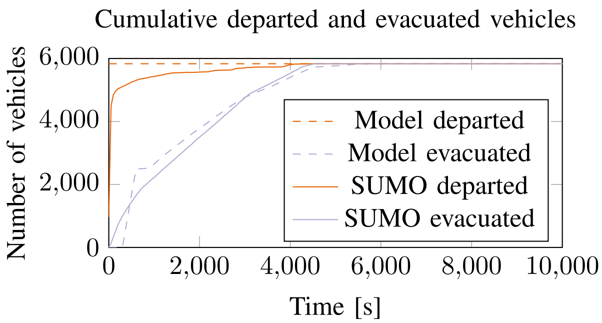

Fig. 4. Cumulative departures and number of evacuated vehicles when all the vehicles are instructed to depart at once. The cumulative curve of evacuated vehicles based on the model reflects the curve of evacuated vehicles observed in the microscopic traffic simulator.

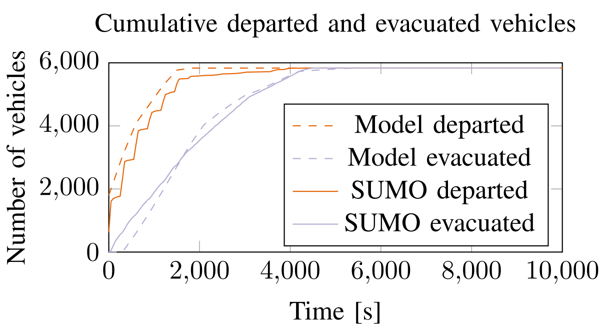

Fig. 5. Cumulative departures and number of evacuated vehicles with a fixed departure rate of 200 vehicles per path every fifth minute. The cumulative curve of evacuated vehicles based on the model reflects the curve of evacuated vehicles observed in the microscopic traffic simulator.

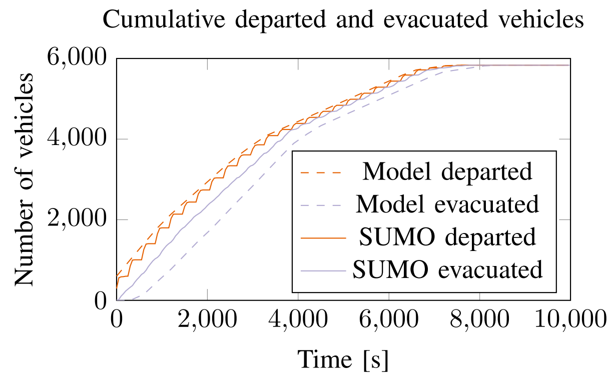

Fig. 6. Cumulative departures and number of evacuated vehicles with a fixed departure rate of 50 vehicles per path every fifth minute.

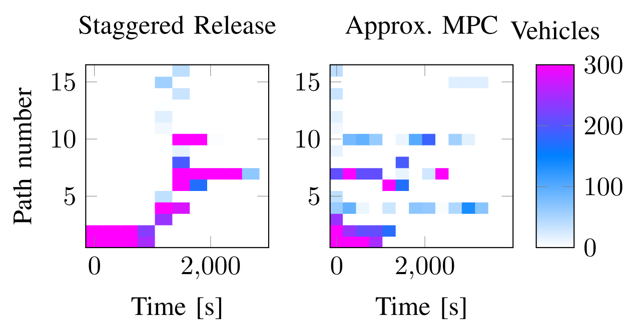

Fig. 7. The number of vehicles released at each time for each path with a staggered release approach and an approximate MPC approach.

waiting once it has been inserted on a road, while the "Average time loss" combines the waiting time and the time lost due to congestion effects, i.e., the driver has to drive with a speed slower than desired. The "Average departure delay" is the time it takes from the time a vehicle is told to depart until it can actually be inserted on the road. This time is included in the computation of the total travel time since it is assumed to be the time that drivers spend in their vehicles.

In the following sections, we present the results for each evacuation strategy in more detail.

1) Fixed departure rate: The cumulative amount of vehicles departed and evacuated for the fixed departure rates are shown in Fig. 4, 5, and 6. In all of the figures, we plot both the trajectories of the dynamical flow network model together with the SUMO simulations to verify that the dynamical network model reflects the dynamics observed in the microscopic traffic simulator.

2) Staggered release: The departure schedule for the staggered release strategy is shown in Fig 7. In Fig. 8 the cumulative number of departed and evacuated vehicles are shown, both for the dynamical network flow model and within SUMO.

3) Approximate MPC: The departure schedule for the approximate MPC is shown in Fig 7. In Fig. 9 the cumulative number of departed and evacuated vehicles are shown, both for the dynamical network flow model and within SUMO.

E. Discussion

By conducting simulations of both the dynamical network flow model (2)–(6) and SUMO simulations of the same evacuation scenarios, we can conclude from the cumulative

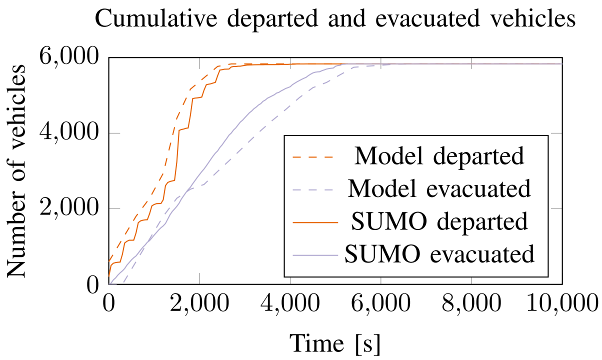

Fig. 8. Cumulative departures and number of evacuated vehicles with a staggered release approach.

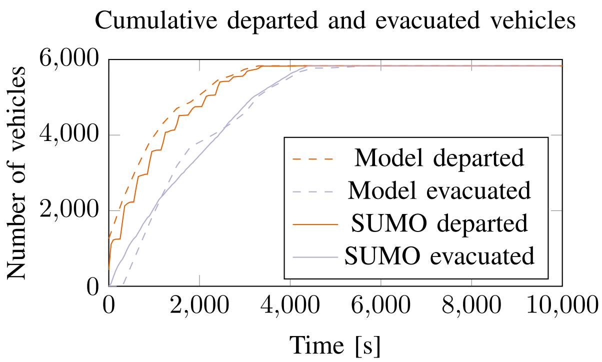

Fig. 9. Cumulative departures and number of evacuated vehicles when the vehicles are released according to the approximate MPC strategy.

curves of evacuated vehicles in Figs. 4, 5, 6, 8, 9 that the dynamical network flow model—despite its simplicity—captures the dynamics of the evacuation process well. Since the dynamical network flow model neglects space constrains when inserting the vehicles, the discrepancy shown in Fig. 4 between the number of departed vehicles in the model and in SUMO simulations is expected.

Both simulations of the model and in SUMO also show that delaying the departure for some of the vehicles does not necessary delay the time it takes to perform the evacuation. This means that there is a possibility for a fraction of the people to wait at home and, e.g., prepare for the evacuation, instead of having to wait on the roads.

From Table IV it is clear that the proposed approximate MPC strategy improves the time it takes to evacuate the city slightly, but reduces the total travel time to almost half. Since none of the strategies managed to largely reduce the time it takes to evacuate, it is likely that the flow capacity constraints in the traffic network are putting a fundamental limit on how quickly the evacuation can be performed.

V. Conclusions

In this paper we have presented a methodology to perform evacuation planning. By constructing an approximate compartmental model for the city traffic, we were able to reduce the total travel time needed to evacuate the city by half through model predictive control techniques. In addition to a relatively modest decrease in total evacuation time, most likely due to capacity constraints in the network, this planning shows that it is possible to give people more time to prepare for the evacuation without delaying the overall evacuation. In the future, we plan to improve integrating the highway dynamics into the model. Since this paper has shown that the relatively simple dynamical flow network model represents the traffic network dynamics well during an evacuation scenario, possibilities open up to develop more advanced control strategies based on the proposed dynamical network flow model.

Acknowledgements

The authors would like to thank Katelyn Iles for her help with constructing a realistic demand scenario for the city of Greensboro, NC.

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