Pricing Parameter Design for Electric Vehicle Charging

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***Abstract*— Pricing models implemented at electric vehicle (EV) charging facilities provide facility operators a means to achieve desirable system-level behavior. Furthermore, a charging facility must meet resource constraints with high levels of confidence. To achieve this, a charging facility operator can tune the pricing model parameters such that resource constraints are met with high confidence. In this paper, we propose an approximate chance-constrained optimization program that enables charging facility operators to set the pricing model parameters in an anticipatory, rather than a reactionary, manner. We present a problem formulation based on two previously developed pricing models and present results from a numerical case study for setting the respective pricing model parameters.**

I. INTRODUCTION

Alternative fuel vehicles such as electric vehicles (EVs) continue to enjoy broad adoption that is placing previously unseen demands on capacity-limited global power and transportation infrastructure [1], [2]. EV charging facilities are resource, i.e., space and power, constrained making it advantageous for charging facility operators to implement operational models to aid them in meeting such constraints. Pricing models are an example of an operational model that provides operators a means for meeting resource constraints.

In practice, a charging facility operator is challenged in having exact knowledge of the arriving users (customers) and their demands. Specifically, users arrive at random times with a collection of random parameters, e.g., charging demand, opportunity cost, etc. Hence, a charging facility must make use of or obtain knowledge of the distribution of these random parameters to effectively implement the pricing models.

Within the EV pricing literature there exist diverse approaches to the problem that utilize knowledge of the user arrivals and their respective parameters in various ways. For example, in [3] the authors study the problem of optimal pricing at an EV charging facility where the charging facility operator and utility provider are separate, independent entities. Here, the authors study the system-wide effects of introducing a large population of EVs with their respective demands and parameters into the power and transportation networks. Similarly, the paper [4] addresses the pricing problem simultaneously from a retail and wholesale perspective where users are faced with an optimization problem to choose their optimal charging behavior, i.e., where and when to charge. In the papers [5], [6] the authors design an online, reservation-based pricing mechanism for EV parking assignments and charge scheduling. None of these papers specifically address the problem of designing parameters for their respective pricing models with considerations for resource constraints.

In this paper, we focus on the design of pricing functions as it relates to their respective parameters. We are specifically interested in charging facilities that offer one of two pricing models: a *defined service level* (DSL) that allows users to choose a rate of charge from a collection of predefined rates and a *prescribed deadline* (PD) model where users choose a charging deadline that is constrained primarily by the maximum allowable charge rate at the charging facility. In these scenarios, users solve an optimization problem to choose either a charging rate in the DSL model or a charging deadline in the PD model which minimizes the total cost to themselves. Both of these models are detailed in [7], [8]. In both the DSL and PD model, the charging facility knows the probability distribution of the user parameters and utilizes this knowledge to compute high-confidence probabilistic bounds that a specified resource level, e.g., number of present users or total charging rate of active users, will not be exceeded.

The papers [7], [8] demonstrate how a charging facility can set the parameters of the pricing function in either the DSL or PD model. There, we perform a reactive, manual tuning of the pricing parameters for both the DSL and PD models to get the desired confidence interval. This reactive approach to setting the charging facility pricing parameters poses a series of challenges. The most notable challenge is that the parameters are tuned reacting to a confidence interval not being as desired. Hence, we are motivated to avoid manually tuning the pricing function parameters. We propose formulating the problem as an approximate chance-constrained optimization program that utilizes an approximation of the confidence intervals as constraints that ultimately yields the desired pricing function parameters.

A. Notation

We denote the positive part of a real number x by $[x]_+ = \max(0, x)$. When considering a collection of independent and identically distributed (i.i.d) random variables indexed by subscripts, we use non-subscript variables when referring to properties that hold for any of the i.i.d random variables. For example, $\mathbb{E}[x]$ is the expectation of each i.i.d random variable x_j.

II. PROBLEM FORMULATION

In this section we present two pricing models that are initially introduced in [7], [8] to study the problem of how an EV charging facility should set the pricing function parameters to meet a particular confidence interval for space and power constraints.

We consider two operating models: in the first model, called the *defined service level model* (DSL), users directly choose from a discrete set of charging rates. In the second model, called the *prescribed deadline model* (PD), users indirectly choose a charging rate by specifying a departure time. In both models, a user's choice is determined by the amount of charge required for their EV, the preferred amount of time they will spend at the local attraction, the prices set by the charging facility, and their impatience factor.

At this facility, a user j arrives at some time a_j (in hr.) with charging demand x_j (in kWh), an impatience factor α_j (in \$/hr.), and a desired (i.e., minimum) amount of time they will spend at the charging location ξ_j (hr.). Throughout the paper we make the following assumption.

Assumption 1 (Users)**.** *User arrivals at the charging facility are a Poisson process with parameter λ (in EVs/hr.). Individual charging demand x_j, the impatience factor α_j, and the time spent at the charging location ξ_j for each user j are random variables which are independent and identically distributed (i.i.d). Additionally, there exists finite $0 < x_{min} < x_{max}$, $0 \leq \alpha_{min} < \alpha_{max}$, and $0 \leq \xi_{min} < \xi_{max}$ such that the distributions of x_j, α_j, and ξ_j are only supported on $[x_{min}, x_{max}]$, $[\alpha_{min}, \alpha_{max}]$, and $[\xi_{min}, \xi_{max}]$, respectively. Furthermore, each x_j is assumed to be a continuous random variable, and α_j may be either discrete or continuous. Lastly, we allow for the possibility that $\xi_{\min} = 0$ and $\mathbb{P}(\xi_j = 0) > 0$ to accommodate the practical special case in which, with nonzero probability, users have no desire to remain at the charging facility. In this case, the distribution of ξ_j is understood to be a generalized probability density function.*

In both models, users balance the need for electric charge with the need for a parking spot for at least their desired time at the local attraction. We formalize these two models in the next two subsections.

A. Defined Service Level (DSL) Model

In the DSL model, the charging facility offers L service levels. Each service level $\ell \in \{1, \dots, L\}$ corresponds to a distinct charging rate $R^\ell > 0$ (in kW) and price $V^\ell > 0$ (in \$/kW) that is the cost per unit energy for the service level. Thus, user j with energy demand x_j pays $x_j V^\ell$ (in \$) to receive a full charge over the time horizon x_j / R^ℓ (in hr.) when choosing service level ℓ. To distinguish the parameters related to the charging facility from those related to the users, the charging facility parameters are upper case and indexed by a superscript, while the parameters for the users are lower case and indexed by a subscript.

Assumption 2 (DSL Model Charging Rates)**.** *Among L service levels offered by the charging facility, a higher charging rate is more costly, i.e., if $R^i > R^k$ then $V^i > V^k$. Moreover, charging rates and prices are distinct so that $R^i \neq R^k$ for all $i \neq k$. Lastly, and without loss of generality, the charging facility's pricing functions are enumerated such that $V^1 < V^2 < \ldots < V^L$ and therefore $R^1 < R^2 < \ldots < R^L$. Define the maximum offered charging rate $R^{max} := R^L$ and $R^{max,\dagger}$ to be the absolute maximum possible charging rate set by a utility provider.*

A user j will remain at a charging facility for the amount of time to completely fulfill their demand x_j. Since user j values their time in excess of the time they want to spend at the charging facility at a rate α_j, they may be willing to pay for a higher service level since it delivers a full charge faster. To this end, the total cost faced by a user arriving at the charging facility is

$$g_\ell(x_j, \alpha_j) = x_j V^\ell + \alpha_j \frac{x_j}{R^\ell} \,. \tag{1}$$

In (1), the first term of the sum, $x_j V^\ell$, is the energy cost to the user resulting from their demand at arrival. The second term of (1), $\alpha_j \left(\frac{x_j}{R^\ell}\right)$, where $\frac{x_j}{R^\ell}$ is the time to charge for a particular service level ℓ, is the cost associated with how much a user values their time. Individual users choose a service level at a charging facility which minimizes their total cost of charging factoring in their impatience. To that end, let $S(x_j, \alpha_j) : [x_{\min}, x_{\max}] \times [\alpha_{\min}, \alpha_{\max}] \to \{1, \dots, L\}$ be defined by

$$S(x_j, \alpha_j) = \underset{\ell \in \{1, \dots, L\}}{\arg\min} \; g_\ell(x_j, \alpha_j) \,. \tag{2}$$

Then, a rational user j chooses service level $S(x_j, \alpha_j)$ in order to minimize their total cost as formalized in the later stated assumption. For notational convenience, we also define the values r_j to be the charging rate and cost per unit of energy chosen by user j after solving (2), i.e., $r_j = R^{S(x_j, \alpha_j)}$.

Assumption 3 (DSL Users are Rational)**.** *Each user chooses a charging rate according to* (2) *and leaves the charging facility once they have satisfied their charging demand. Thus, user j occupies a charger at the facility during the time interval $[a_j, a_j + x_j / r_j]$.*

The DSL model allows for a charging facility that offers multiple discrete charging levels as is seen in existing charging infrastructure which is currently divided into three charging levels [9]. In some cases, it may be more convenient for the user to provide a deadline by which they expect to receive a full charge. Such pricing schemes have indeed been implemented in practice [10]. We develop a deadline model in the following subsection.

B. Prescribed Deadline (PD) Model

As in the DSL model, in the PD model a user j arrives with charging demand x_j, a desired time at the location ξ_j, and an impatience factor α_j. However, in the PD model, the user j chooses a charging deadline u_j rather than a discrete charging rate. The charging facility broadcasts a single pricing function $P(x_j, u_j)$ that constitutes the financial cost to a user receiving charge x_j over the deadline u_j. Then,

$$C(x_j, u_j, \alpha_j, \xi_j) = P(x_j, u_j) + \alpha_j (u_j - \xi_j) \tag{3}$$

is the total cost faced by a user j who arrives with demand x_j, impatience factor α_j, planned time at location ξ_j, and who chooses a charging deadline $u_j \geq \xi_j$. Hence, (3), includes a penalty for choosing a charging deadline u_j greater than their desired time at a location ξ_j at a rate α_j. We note that it is possible that $P(x_j, u_j)$ also contains a similar time penalty. A rational user j chooses their charging deadline according to

$$u_j \in \underset{u \geq \xi_j}{\arg\min}\, C(x_j, u, \alpha_j, \xi_j)\,. \tag{4}$$

In (3), we see that in addition to paying a price to charge as a function of the demand and chosen deadline, a user faces an opportunity cost which is a function of their impatience and how much time beyond their desired time, ξ_j, they spend at the charging facility.

Assumption 4 (PD Users are Rational)**.** *Each user chooses a charging deadline according to* (4) *and leaves the charging facility at the chosen deadline. Thus, user j occupies a charger at the facility during the time interval* $[a_j, a_j + u_j]$.

There are physical limitations on the charging facilities such as a maximum charging rate allowable per user since we explore the problem of resource-constrained charging facilities. This point is formalized in the following assumption.

Assumption 5 (PD Model Charging Rates)**.** *The pricing function $P(x_j, u_j)$ is such that there exists an upper bound $R^{max,\dagger}$ on the charging rate for any user solving* (4) *under the PD model,* i.e., *$R^{max,\dagger} \geq r_j$, where $r_j = x_j/u_j$, for all users j when u_j is chosen according to* (4)*. Moreover, the charging facility provides electric power at the constant rate r_j over the charging time horizon u_j for each user j.*

Remark 1. *In the PD model, note that the charging deadline u_j, and therefore also the charging rate $r_j = x_j/u_j$, is a continuous random variable. This contrasts with the DSL model where r_j is a discrete random variable.*

There exist many candidate functions that can be utilized as pricing functions. The following is such an example.

Example 1. *Consider the pricing function*

$$P(x_j, u_j) = x_j \left(D(u_j - \omega)^2 + B\right)\,, \tag{5}$$

where D is the surge price (in \$/kWh-hr.^{2}), ω is the time offset parameter (in hr.), and B is the base price (in \$/kWh).Then, from (4)*, a user j chooses deadline*

$$u_j \in \underset{u \geq \xi_j}{\arg\min}\; x_j \left(D(u - \omega)^2 + B\right) + \alpha_j(u - \xi_j)\,. \tag{6}$$

The term $(u_j - \omega)^2$ penalizes a user for choosing a deadline less than or greater than ω at a surge price rate D.

Since (3) *substituted with* (5) *is convex in u, the minimizer is unique and available in closed-form so that user j will choose deadline*

$$u_j := u^* = \max\left\{\xi_j, \frac{-\alpha_j}{2Dx_j} + \omega\right\}. \tag{7}$$

As previously mentioned, we operate under Assumption 5, i.e., $R^{max} \geq x_j/u_j$ must hold. Interpreting R^{max} as an a priori *fixed limit, the charging facility must then choose parameters D, B, and ω to satisfy Assumption 5. Algebraic manipulations combined with reasoning when the maximum is attained lead to the fact that $\omega > x_{max}/R^{max}$ and*

$$D > \left[\max_{x_j \in [x_{min}, x_{max}]} \frac{\alpha_{min} R^{max,\dagger}}{2\omega x_j R^{max,\dagger} - 2x_j^2}\right]_+ . \tag{8}$$

Remark 2. *In the DSL model the user will remain at the charging location for x_j/r_j where $r_j = R^{S(x_j,\alpha_j)}$. In the PD case, a user selects a deadline u_j and the appropriate rate is set that fulfills the charging demand exactly at the deadline time.*

Next, we formally introduce the problem statement for the charging facility which lays the foundation for the main result for both the DSL and PD models.

C. Guarantees on Charging Facility Capacity Limits

Let the set of present users at the charging facility at time t be defined as

$$N(t) = \begin{cases} \{i : t \in [a_i, a_i + x_i/r_i]\} & \text{if DSL} \\ \{i : t \in [a_i, a_i + u_i]\} & \text{if PD} \end{cases}$$

and let $\eta(t) = |N(t)|$ be the cardinality of the set of present users. Note that the set of actively charging users is the same as the set of present users at the charging facility. Then,

$$Q(t) = \sum_{i \in N(t)} r_i$$

is the total charging rate at time t for all actively charging users, i.e., the charging facility's total power consumption. Note that $r_i = x_i/u_i$ in the summation for the PD model.

Theorem 1. *(Theorem 1 from [8]) Consider a charging facility operating under the DSL model (resp., PD model) with Assumptions 1, 2, and 3 (resp., Assumptions 1, 4, and 5). Let*

$$\theta = \begin{cases} x/r & \textit{if DSL model} \\ u & \textit{if PD model,} \end{cases}$$

i.e., the model dependent random time spent at the charging facility for each user. Given any $\mathcal{M} \geq 0$ number of users and $\mathcal{R} \geq 0$ total charging rate, the following statements hold at steady state for any time t:

1) *With confidence $1 - \delta(\mathcal{M})$, where*

$$\delta(\mathcal{M}) = \begin{cases} \exp\left(\frac{-(\mathcal{M} - \lambda\,\mathbb{E}[\theta])^2}{2\left(\lambda\,\mathbb{E}[\theta] + \frac{(\mathcal{M} - \lambda\,\mathbb{E}[\theta])}{3}\right)}\right) & \textit{if } \mathcal{M} > \lambda\,\mathbb{E}[\theta] \\ 1 & \textit{otherwise,} \end{cases}$$

the number of users will not exceed $\mathcal{M}$, i.e., $\mathbb{P}(\eta(t) < \mathcal{M}) \geq 1 - \delta(\mathcal{M})$.

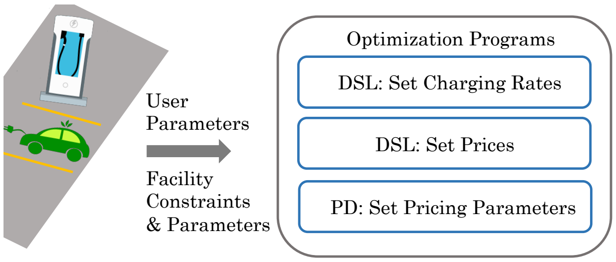

Fig. 1. A charging facility operates under one of two models: a defined service level (DSL) model where users choose a charge rate from a finite collection of choices and a prescribed deadline (PD) model where users set a charging deadline upon which they will receive a complete charge. Here, a charging facility designs the pricing function parameters such that it has high confidence level in maintaining its space and power constraints. In the DSL and PD model, the charging facility solves one of three approximate chance-constrained optimization program to obtain set the pricing parameters.

2) *With confidence* $1-\gamma(\mathcal{R})$*, where*

$$\gamma(\mathcal{R}) = \begin{cases} \min\left\{1, \sum_{m=\left\lceil \frac{\mathcal{R}}{R^{max}} \right\rceil}^{\left\lfloor \frac{\mathcal{R}}{\mathbb{E}[r]} \right\rfloor} \exp\left(\frac{-(\mathcal{R}-m\,\mathbb{E}[r])^2}{2\left(m\,\mathbb{E}[r^2]+\frac{R^{max}(\mathcal{R}-m\,\mathbb{E}[r])}{3}\right)}\right) \right. \\ \left. \times \mathbb{P}\big(\eta(t)=m\big) + \delta\left(\left\lfloor \frac{\mathcal{R}}{\mathbb{E}[r]} \right\rfloor\right) \right\}, \quad \text{if } \mathcal{R} > \lambda\,\mathbb{E}[\theta]\,\mathbb{E}[r] \\ 1, \quad \text{otherwise,} \end{cases}$$

the total charging rate for all active users will not exceed $\mathcal{R}$*, i.e.,* $\mathbb{P}\left(Q(t) < \mathcal{R}\right) \geq 1-\gamma(\mathcal{R})$.

The full proof of Theorem 1 is presented in [8].

III. Methodology

In this section, we present the approximate chance-constrained optimization programs that we solve to set the pricing parameters for both the DSL and PD models. For the DSL model, we consider the case where a charging facility is interested in setting the service level charging rates while minimizing the expected time users spend actively charging, $\mathbb{E}[\theta]$, and also the case when the charging facility is interested in setting the price per unit energy, V^ℓ, of the offered service levels while maximizing their weighted sum, $w_1V^1 + \cdots + w_LV^L$, across all service levels. For the PD model, we consider the case where a charging facility has a pricing function $P(x_j, u_j)$ as in (5) where the charging facility seeks to set the parameters D and ω while maximizing $\mathbb{E}[\theta]$. The following numerical studies are solved using MATLAB's Optimization Toolbox.[1]

A. Setting Charging Rates and Prices for the DSL Model

Given the randomness of user arrivals and their respective parameters the guarantees on the resource constraints are probabilistic in nature. Hence, a charging facility will set the L service level charging rates subject to resource utilization confidence interval constraints. We can construct an optimization problem to set the charging rates; however, since some constraints are probabilistic, we are specifically solving a chance-constrained optimization program.

In general, chance-constrained optimization problems are difficult to solve because there often is not a readily available, closed form expression for probabilistic constraints such as $\mathbb{P}\left(\eta(t) < \mathcal{M}\right)$ and $\mathbb{P}\left(Q(t) < \mathcal{R}\right)$ [11]. To address this problem we formulate an approximate chance-constrained optimization problem. Instead of finding a direct closed form expression of $\mathbb{P}\left(\eta(t) < \mathcal{M}\right)$ and $\mathbb{P}\left(Q(t) < \mathcal{R}\right)$ we find an approximation for these values that is used as a constraint in the optimization program, and hence, leading to an approximate chance-constrained optimization program. Finding approximations of these probabilities can be difficult depending on the nature of the problem. Fortunately, we can make use of the expressions of Theorem 1 as approximations. Noting the results of Theorem 1, we realize that $\mathbb{P}\left(\eta(t) < \mathcal{M}\right) \geq 1-\delta(\mathcal{M})$ and $\mathbb{P}\left(Q(t) < \mathcal{R}\right) \geq 1-\gamma(\mathcal{R})$ and we utilize the respective lower bounds as constraints in the chance-constrained optimization program.

Here, we are specifically interested in a charging facility operating under the DSL model and interested in setting the service level charge rates while minimizing the expected time users are actively charging, $\mathbb{E}[\theta]$. For the DSL model, by definition, $\mathbb{E}[\theta] = \mathbb{E}[x/r] = \mathbb{E}[x]\,\mathbb{E}[1/r] = \mathbb{E}[x]\sum_{i=1}^{L} p(r_i)\frac{1}{R^i}$ where $p(r_i)$ in the summation is the PMF of r_j, i.e., the rate choice of user j. The expression $p(r_i)$ is defined in the papers [7], [8] for the special case when there is no parking fee and $\xi_j = 0$ for all j. The PMF of r_j is difficult to solve in general and is challenging to include in the objective of the approximate chance-constrained optimization program. However, note that each $0 \leq p(r_i) \leq 1$ in the summation when considered in the context of an optimization objective is essentially a weighting on each $1/R^i$. Given the difficulties of using the $p(r_i)$ and noting the $\mathbb{E}[x]$ is simply a constant when the charge rates are the decision variable, we choose an alternative objective that is an unweighted version of the original objective. This leads to the following optimization program.

Program 1 (Optimization to Set DSL Charge Rates)**.**

$$\begin{aligned} \min_{R^m} \quad & \sum_{m=1}^{L} \frac{1}{R^m} \\ \textit{s.t.} \quad & R^i > R^k + \epsilon_{R_{ik}} \quad \textit{for all } i > k \\ & 0 < R^i < R^{max,\dagger} \quad \textit{for } i \in \{1,\ldots,L\} \\ & 1-\delta(\mathcal{M}) \geq \epsilon_\delta \\ & 1-\gamma(\mathcal{R}) \geq \epsilon_\gamma \end{aligned}$$

Program 1 formalizes the prior statements regarding a charging facility operator setting the service level charge rates. In Program 1, $\epsilon_{R_{ik}}$ represents a user set parameter of the minimum spacing between charge rate i and k, $0 \leq \epsilon_\delta \leq 1$ is the confidence value an operator seeks for the number of present or actively charging users, and $0 \leq \epsilon_\gamma \leq 1$ is the confidence value an operator seeks for the total charge rate of actively charging users.

[1]The code for this case study is available at `https://github.com/gtfactslab/setchargingparameters`

Given $\delta(\mathcal{M})$ and $\gamma(\mathcal{R})$ in Theorem 1 we realize Program 1 is non-convex. However, since we are addressing the problem in the context of EV charging there are real-world limitations on the service level charge rates, and hence, the domain for each R^i for $i \in \{1, \ldots, L\}$ is bounded. This allows a charging facility operator to solve this optimization program by using generalized nonlinear solvers such as MATLAB's FMINCON() along with sampling techniques on the initial conditions. This is detailed in Section IV when discussing the numerical study.

A charging facility may also be interested in setting the prices $V^\ell < V^{\max}$ for all ℓ where $V^{\max}$ is the maximum price at the charging facility. To achieve this a charging facility operator can set prices according to the following optimization program.

Program 2 (Optimization to Set DSL Prices)**.**

$$\begin{aligned} \max_{V^m} \quad & \sum_{m=1}^{L} w_m V^m \\ \textit{s.t.} \quad & V^i > V^k + \epsilon_{V_{ik}} \quad \textit{for all } i > k \\ & 0 < V^i < V^{\max} \quad \textit{for } i \in \{1, \ldots, L\} \\ & 1 - \delta(\mathcal{M}) \geq \epsilon_\delta \\ & 1 - \gamma(\mathcal{R}) \geq \epsilon_\gamma \end{aligned}$$

Program 2 formalizes a charging facility operator setting the service level prices according to series of constraints. In Program 2, similar to Program 1, $\epsilon_{V_{ik}}$ represents a user set parameter of the minimum spacing between pricing of service level i and k, $0 \leq \epsilon_\delta \leq 1$ is the confidence value an operator imposes on the number of actively charging users, and $0 \leq \epsilon_\gamma \leq 1$ is the confidence value an operator imposes for the total charge rate of actively charging users. Next, we present the scenario when a charging facility operating under the PD model is interested in setting its pricing parameters.

B. Setting Pricing Parameters for the PD Model

Consider the case when a charging facility is operating under the PD model and $P(x_j, u_j)$ is as in Example 1. The two pricing parameters of interest are the surge price D and the time offset parameter ω whose domains are restricted as detailed in Example 1. In addition to the lower bound on D detailed in Example 1 a charging facility imposes a maximum surge price D_{max}. Note that D_{max} is driven by a desire to limit the surge price users can face rather than physical charging facility limitations as is the case for the lower bound of the surge price D as is detailed in Example 1. Thus, we utilize the fact that $\mathbb{P}\left(\eta(t) < \mathcal{M}\right) \geq 1 - \delta(\mathcal{M})$ and $\mathbb{P}\left(Q(t) < \mathcal{R}\right) \geq 1 - \gamma(\mathcal{R})$ from Theorem 1 to formulate an approximate chance-constrained optimization program.

The objective of this program is to maximize the expected time users spend actively charging at the charging facility. In the PD model a charging facility particularly sensitive to the total charge rate of actively charging users may use this objective, as opposed to the minimization in Program 1.

Specifically, the objective would be $\max \mathbb{E}[u_j]$ where u_j is as defined in (7). However, using this expression as the objective of the optimization program is challenging. As a result, after some algebraic reasoning, we create an alternative objective $\max \frac{1}{D} + \omega$ to attempt to mimic the behavior of (7). While admittedly it is not an exact account, we attempt to maintain the inverse relationship between the surge price D and the deadline u_j while also maintaining the positive correlation between ω and u_j.

Taking into consideration the domain restrictions of the pricing function (5) and the approximate confidence interval formulas, we can formulate the following optimization program for the PD model.

Program 3 (Optimization to Set PD Model Surge Price & Time Offset Parameter)**.**

$$\begin{aligned} \max_{D,\omega} \quad & \frac{1}{D} + \omega \\ \textit{s.t.} \quad & \omega > x_{max}/R^{max} \\ & D_{max} > D > \left[\max_{x_j \in [x_{min}, x_{max}]} \frac{\alpha_{min} R^{max}}{2\omega x_j R^{max} - 2x_j^2} \right]_+ \\ & 1 - \delta(\mathcal{M}) \geq \epsilon_\delta \\ & 1 - \gamma(\mathcal{R}) \geq \epsilon_\gamma \end{aligned}$$

Program 3 formalizes the prior statements regarding the approximate probability constraints and the constraints on the pricing function parameters D and ω. In the next section we present the results of solving the 3 aforementioned optimization programs.

IV. Numerical Study Discussion

In this section we present the implementations of Program 1–3 and discuss the numerical results from each optimization program implementation.

First, we construct a charging facility operating under the DSL pricing model. We specifically consider the case where a charging facility is attempting to set $L = 3$ service level charge rates with pricing functions of the form of (1). Each service level has its own prices per unit energy where $V^1 = 0.20$ \$/kWh, $V^2 = 0.25$ \$/kWh, and $V^3 = 0.30$ \$/kWh. We consider the EV charging problem within the context of physical constraints, and hence, the maximum charge rate of the facility is $R^{\max} = 40$kW. Furthermore, a charging facility desires a minimum charge rate spacing of $\epsilon_{R_{21}} = 4$kW and $\epsilon_{R_{32}} = 5$kW. The imposed approximate chance constraints for the resource levels $\mathcal{M} = 60$ and $\mathcal{R} = 1000$ kW are $\epsilon_\delta = 0.85$ and $\epsilon_\gamma = 0.85$, respectively. Lastly, we assume $x_j \sim \mathcal{U}(5, 60)$ where $\mathcal{U}(\cdot)$ denotes a uniform distribution and α_j is a discrete random variable taking the values 5, 10, and 15 with equal probability of $1/3$ each.

Given these parameters, a charging facility operator sets the $L = 3$ charging rates by solving Program 1. Since Program 1 is non-convex the locally optimal feasible solution, if there is one, will sometimes vary depending on the initial condition of the solver. To handle this issue we resort to a sampling of initial conditions to find the optimal set of charging rates amongst a collection of feasible charging rates. We first restrict the sampling technique to integer-valued initial conditions greater than some value $R_0^{\min}$ and less than the maximum allowable charge rate at the charging facility $R^{\max,\dagger}$. To initialize Program 1, the charging facility chooses 3 initial values from a range of $[R_0^{\min}, R^{\max,\dagger}]$ where $R_0^{\min} > 0$. Then, a charging facility orders these values such that Assumption 2 is respected in the initial conditions. This means that at most there are $\binom{n}{3}$ initial conditions where $n = R^{\max,\dagger} - R_0^{\min} + 1$. Note that $R_0^{\min}$ is the minimum threshold on the initial conditions of the optimization program and is not an imposed minimum on the allowable rate at the charging facility while $R^{\max}$ is a maximum on the allowable rate for both the DSL and PD model. The value $R_0^{\min}$ is considered to reduce the number of initial conditions sampled. Given these observations, we solve Program 1 and present the top 3 results in Table I.

TABLE I

NUMERICAL RESULTS OF PROGRAM 1

Initial Condition (kW)	Final Rates	Cost
$\mathbf{38, 39, 40}$	$\mathbf{24.72, 33.22, 39.85}$	$\mathbf{0.0956}$
$35, 38, 39$	$24.76, 32.91, 39.75$	0.0959
$35, 39, 40$	$24.57, 32.58, 39.93$	0.0964

The second part of the DSL numerical study addresses the problem of setting the service level prices V^ℓ by solving Program 2. We consider the case where $L = 3$, and the respective charge rates are $R^1 = 20$kW, $R^2 = 25$kW, and $R^3 = 30$kW, and we use the weights $w_i = 1$ for $i = 1, \ldots, L$. Furthermore, we take $\epsilon_{V_{21}} = .05$, and $\epsilon_{V_{32}} = .04$ as the minimum spacing between the respective service level prices. Additionally, $\mathcal{M} = 30$, $\epsilon_\delta = 0.30$, $\mathcal{R} = 800$kW, and $\epsilon_\gamma = 0.75$. Lastly, $x_j \sim \mathcal{U}(5, 60)$ and α_j is a discrete random variable taking the values 5, 15, and 20 with equal probability. As in the Program 1 case, we sample a series of initial conditions to find the optimal set of service level prices amongst the collection of feasible prices. Given these parameters, we solve Program 2 and present the results in Table II.

TABLE II

NUMERICAL RESULTS OF PROGRAM 2

Initial Condition ($/kWh)	Final Prices	Objective Value
$\mathbf{0.41, 0.45, 0.49}$	$\mathbf{0.3963, 0.4463, 0.4932}$	$\mathbf{1.3358}$
$0.43, 0.45, 0.49$	$0.3935, 0.4435, 0.4973$	1.3344
$0.42, 0.45, 0.49$	$0.3944, 0.4444, 0.4952$	1.3339

The last part of the numerical study addresses the problem of a charging facility operating under the PD model with a pricing function of the form of (5). Here, a charging facility sets the surge price D and the desired time offset parameter ω subject to the approximate probabilistic resource constraints for specified values of $\mathcal{M}$ and $\mathcal{R}$. Specifically, we consider the case where $\mathcal{M} = 100$, $\mathcal{R} = 1400$, $\epsilon_\delta = 0.30$, and $\epsilon_\gamma = 0.75$. Lastly, we take $x_j \sim \mathcal{U}(10, 100)$, $\xi_j \sim \mathcal{U}(0, 2.5)$ and $\alpha_j \sim \mathcal{U}(2, 10)$. To set the parameters D and ω, the charging facility must solve Program 3. These results are presented in Table III.

TABLE III

NUMERICAL RESULTS OF PROGRAM 3

Init. Cond. ($/kWh-hr.^{2}, hr.)	Final Param.	Objective Val.
$\mathbf{2.00, 3.00}$	$\mathbf{0.4542, 4.9759}$	$\mathbf{7.1778}$
$2.00, 2.50$	$0.5976, 4.8174$	6.4907
$2.25, 2.50$	$1.0606, 4.6371$	5.5800

V. CONCLUSION

In this paper, we study the problem of setting the pricing function parameters at a charging facility that operates on either the DSL model where users choose a charge rate that minimizes the cost to themselves from a finite collection of charge rates or the PD model where users choose a charging deadline that minimizes the total cost to themselves. In the DSL model, we focus on a charging facility setting the appropriate service level charge rates and prices. In the PD model, we consider a particular pricing function example and focus on setting the two respective pricing function parameters. For both models, we construct approximate chance-constrained optimization programs that make use of a closed-form expression of the confidence level for both space and power constraints at the EV charging facility. While the optimization problems are non-convex and difficult to solve in general, we address the non-convexities of the optimization problems by utilizing sampling methods that take advantage of the bounded domain to find a solution to the problem. We present a numerical study that illustrates the efficacy of the numerical techniques.

REFERENCES

[1] H. Das, M. Rahman, S. Li, and C. Tan, "Electric vehicles standards, charging infrastructure, and impact on grid integration: A technological review," *Renewable and Sustainable Energy Reviews*, vol. 120, p. 109618, 2020.

[2] T. Gnann, S. Funke, N. Jakobsson, P. Plötz, F. Sprei, and A. Bennehag, "Fast charging infrastructure for electric vehicles: Today's situation and future needs," *Transportation Research Part D: Transport and Environment*, vol. 62, pp. 314–329, 2018.

[3] M. Alizadeh, H.-T. Wai, M. Chowdhury, A. Goldsmith, A. Scaglione, and T. Javidi, "Optimal pricing to manage electric vehicles in coupled power and transportation networks," *IEEE Transactions on Control of Network Systems*, vol. 4, no. 4, pp. 863–875, 2016.

[4] M. Alizadeh, H.-T. Wai, A. Goldsmith, and A. Scaglione, "Retail and wholesale electricity pricing considering electric vehicle mobility," *IEEE Transactions on Control of Network Systems*, vol. 6, no. 1, pp. 249–260, 2018.

[5] N. Tucker, B. Ferguson, and M. Alizadeh, "An online pricing mechanism for electric vehicle parking assignment and charge scheduling," in *2019 American Control Conference (ACC)*. IEEE, 2019, pp. 5755–5760.

[6] N. Tucker and M. Alizadeh, "An online admission control mechanism for electric vehicles at public parking infrastructures," *IEEE Transactions on Smart Grid*, vol. 11, no. 1, pp. 161–170, 2019.

[7] C. Santoyo, G. Nilsson, and S. Coogan, "Multi-level electric vehicle charging facilities with limited resources," in *IFAC World Congress*, 2020.

[8] ——, "Resource aware pricing for electric vehicle charging," *arXiv preprint arXiv:2009.10771*, 2020.

[9] A. F. D. Center, *Vehicle Charging*, https://afdc.energy.gov/fuels/electricity_infrastructure.html.

[10] N. R. E. Laboratory, *Electric Vehicle Pricing and Payments*, https://www.nrel.gov/about/ev-charging-stations-pricing.html.

[11] A. Shapiro, D. Dentcheva, and A. Ruszczyński, *Lectures on stochastic programming: modeling and theory*. SIAM, 2014.