Sensitivity of Electric Vehicle Charging Facility Occupancy to Users' Impatience

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Abstract— **In this paper, we consider an electric vehicle charging facility that offers various levels of service, i.e., charging rates, for varying prices such that rational users choose a level of service based on their value of time, also called impatience. In particular, we characterize the sensitivity of the expected number of users, i.e., occupancy, at the facility to the probability distribution of users' impatience. We first provide an upper bound for the difference between the expected occupancy under any two different distributions on users' impatience. Next, we consider the case when the users' impatience are discrete random variables, and we study the sensitivity of the expected occupancy to the probability masses and attained values of the random variables. We show that the expected occupancy varies linearly with respect to the probability masses and is piecewise constant with respect to the attained values. These results suggest how the facility operator might design prices such that the expected occupancy does not vary much under small changes in the distribution of users' impatience, which is generally difficult to characterize accurately from data. We demonstrate this idea via examples.**

I. Introduction

Improved affordability of electric vehicles (EVs) has catalyzed EV adoption trends such that by 2040 it is projected that 58% of global new vehicle sales will be EVs [1]. This surging adoption of EVs places increased demands on EV charging facilities which are generally resource-constrained and necessitates the study of the system-level behavior of limited resources such as charging space. To study such behavior, there have been efforts to study the problem from various perspectives [2], [3], [4] including a pricing model approach [5].

Additional past works have focused on studying EV charging within a utility provider framework where the utility provider and charging facility are separate entities whose pricing engenders specific system-wide behavior [6]. Furthermore, in the paper [7] the authors present a location-based pricing scheme and analyze its effects on system-wide congestion. The paper [8] considers a spatiotemporal model for rapid charging facilities where the authors utilize a queuing theoretic model to predict charging demand when the user arrival rate is not known a priori. The paper [9] models EV charging within a queuing framework to to formulate an equilibrium assignment model. Each of these previous works relies on specific modeling assumptions; however, none of the aforementioned papers perform a sensitivity analysis of their respective models to understand how their models perform when the known information is erroneous. The need for understanding EV charging model sensitivity is the main motivator for the present work.

In the present paper, we focus on the model that is originally presented in the paper [10]. Here, users arrive at random times with a collection of random parameters whose respective distributions are assumed to be known. Specifically, the user parameters are the user's energy demand and their value of time. Furthermore, in this model, users are presented with a discrete and finite collection of charging rates and energy prices from which they choose the charging rate and price that minimizes the total cost to themselves. Each charging rate is associated with a specific service level pricing function that defines the total cost to the user of choosing a particular charging rate as a function of the user's demand parameters. Although this paper is applying the user choice model to analyze electric vehicle charging facilities, cloud computing [11] and ride-sharing services [12] among others are examples of when users can face similar trade-off decisions when paying for a service [13].

An appropriately designed pricing scheme allows the charging facility operator to manage the limited resources. Since charging facilities face random demands it is challenging to predictively analyze the exact space usage at any given time. Instead, a charging facility may study the expected occupancy and the expected total charge rate of the actively charging users. Using a pricing scheme of [5], [10] one can derive explicit formulas for these expected values under the assumption that the distributions of user arrivals and their parameters are known. Specifically, it is seen in the papers [5], [10] that the computation of these system-wide expected values is dependent on knowing the distribution of user's value of time.

In practice, given the wide availability of data on EVs and consumer habits, it is reasonable to assume that a charging facility can obtain good estimates of quantities that can be explicitly measured such as user arrival times, and user's energy demands. However, to compute the aforementioned system-wide attributes a charging facility must also know the distribution of the user's value of time to characterize system-wide behavior. While recent studies attempt to quantify such behavioral factors [14] in practice, obtaining an accurate characterization remains challenging.

We are motivated to quantify and analyze the sensitivity

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of the expected occupancy to mischaracterizations of user's value of time distribution. To do so, we derive an expression that describes how the expected occupancy at a charging facility changes with a given characterization of the user's value of time. Furthermore, we derive a worst-case error bound for the expected occupancy. We specifically consider the cases when a user's impatience can be placed into a finite number of categories, i.e., the impatience factor is a discrete random variable. To the best of our knowledge, this is the first work performing a sensitivity analysis of a pricing model implemented at an EV charging facility.

This paper is organized as follows: Section II presents the model formulation for the pricing model. Section III presents the sensitivity analysis results. Section IV details a numerical study and Section V presents the conclusions.

A. Notation

For an indexed set of variables $\{x^k\}$, we let $\Delta^i_j x$ denote the difference between the variable with index i and j, i.e., $\Delta^i_j x = x^i - x^j$. When considering a collection of independent and identically distributed (i.i.d) random variables indexed by subscripts, we use non-subscript variables when referring to properties that hold for any of the i.i.d random variables. For example, $\mathbb{E}[x]$ is the expectation of each i.i.d random variable x_j.

II. Problem Formulation

In this section, we present a pricing model for EV charging that was initially introduced in [5], [10]. We consider a *defined service level model* where users directly choose from a discrete set of charging rates and prices upon arrival at the charging facility; a user pays a higher price for a faster charge rate. A rational user chooses a charge rate depending on the amount of charge required for their EV, the prices and rates set by the charging facility, and their impatience factor, i.e., their value of time. In this paper, we assume that users generally prefer to minimize the cost to themselves and depart the facility immediately upon receiving a full charge.

At this facility, a user j arrives at some time τ_j (in hr.) with charging demand x_j (in kWh), and an impatience factor α_j (in \$/hr.). Throughout the paper, we make the following assumption about the aforementioned variables.

Assumption 1 (Users)**.** *User arrivals at the charging facility are a Poisson process with parameter λ (in EVs/hr.). Individual charging demand x_j, and the impatience factor α_j for each user j are random variables which are independent and identically distributed (i.i.d). In particular, x_j is a continuous random variable with support $[x_{min}, x_{max}]$ for some $0 < x_{min} < x_{max}$. Furthermore, α_j is a discrete random variable with M possible values whose probability mass function $p_A(\alpha; p, a)$ has a probability mass vector $p = [p_1, \dots, p_M]^\top$ corresponding to the impatience value vector $a = [a_1, \dots, a_M]^\top$ such that $\mathbb{P}(\alpha_j = a_i) = p_A(a_i; p, a) = p_i$ for each i.*

When using the probability operator $\mathbb{P}(\cdot)$ for α_j it is understood that this probability is computed with some probability mass vector p and impatience category vector a. By assuming α_j are i.i.d discrete random variables, we assume the population of users is divided into a finite number of impatience categories; for example, each user might be either patient with a low value of α or impatient with a high value of α.

The charging facility offers L service levels. Each service level $\ell \in \{1, \dots, L\}$ corresponds to a distinct charging rate $R^\ell > 0$ (in kW) and price $V^\ell > 0$ (in \$/kWh) that is the cost per unit energy for the service level. Thus, user j with energy demand x_j pays $x_j V^\ell$ (in \$) to receive a full charge over the time horizon x_j/R^ℓ (in hr.) when choosing service level ℓ. To distinguish the parameters related to the charging facility from those related to the users, the charging facility parameters are upper case and indexed by a superscript, while the parameters for the users are lower case and indexed by a subscript j.

Assumption 2 (Model Charging Rates)**.** *Among L service levels offered by the charging facility, a higher charging rate is more costly, i.e., if $R^i > R^k$ then $V^i > V^k$. Moreover, charging rates and prices are distinct so that $R^i \neq R^k$ for all $i \neq k$. Lastly, and without loss of generality, the charging facility's pricing functions are enumerated such that $V^1 < V^2 < \dots < V^L$ and therefore $R^1 < R^2 < \dots < R^L$.*

A user can therefore pay less by choosing a slower charge rate but must balance this with their impatience. In particular, the total cost faced by a user arriving at the charging facility with impatience factor α_j, charging demand x_j, and who chooses service level ℓ, is

$$g_\ell(x_j, \alpha_j) = x_j V^\ell + \alpha_j \frac{x_j}{R^\ell}\,. \tag{1}$$

In (1), the first term of the sum is the energy cost to the user and the second term is the cost associated with how much a user values their time. Individual users choose a service level at a charging facility which minimizes their total cost of charging factoring in their impatience. To that end, let $S(x_j, \alpha_j) : [x_{\min}, x_{\max}] \times \{a_1, \dots, a_M\} \to \{1, \dots, L\}$ be defined by

$$S(x_j, \alpha_j) = \underset{\ell \in \{1, \dots, L\}}{\arg\min}\; g_\ell(x_j, \alpha_j)\,. \tag{2}$$

Then, a rational user j chooses service level $S(x_j, \alpha_j)$ in order to minimize their total cost as formalized in the later stated assumption.

For notational convenience, we also define the values r_j to be the charging rate and cost per unit of energy chosen by user j after solving (2), i.e., $r_j = R^{S(x_j, \alpha_j)}$. Observe that the user charging times x_j/r_j, being uniquely determined by x_j and α_j, constitute a collection of independent and identically distributed random variables. Furthermore, this means the time a user spends at the charging location is x_j/r_j where this is the time for a user to receive a full charge based on their chosen service level.

Assumption 3 (Users are Rational)**.** *Each user chooses a charging rate according to* (2) *and leaves the charging* *facility once they have satisfied their charging demand. Thus, user j occupies a charger at the facility during the time interval $[\tau_j, \tau_j + x_j/r_j]$.*

Charging facilities are concerned with adhering to user capacity restrictions. Let the occupancy set at the charging facility at time t be defined as

$$N(t) = \{i : t \in [\tau_i, \tau_i + x_i/r_i]\}$$

and let $\eta(t) = |N(t)|$ be the cardinality of the occupancy set. Since users are assumed to leave immediately after receiving a full charge, $N(t)$ is also the set of actively charging users.

III. Main Results

The capacity constraint on EV charging facilities makes it imperative for charging facility operators to have an accurate-as-possible estimate of the expected occupancy. At a given charging facility with pricing functions of the form of (1), users arrive at random times with random parameters to ultimately make a service level choice that minimizes the cost to themselves by solving (2).

To obtain an expression for the occupancy, recall Lemma 1 below from the paper [5], which details how a user j arriving with charging demand x_j and impatience factor α_j chooses a specified service level while facing pricing functions of the form of (1). Prior to presenting Lemma 1 we introduce Assumption 4 which restricts the possible energy price and charging rate values set by the charging facility.

Assumption 4. *A charging facility offers L service levels with price per unit energy V^ℓ and charging rate R^ℓ according to Assumption 2 such that $\Delta_k^i V/\Delta_i^k \bar{R} \neq a_m$ for all k, i for any m.*

Assumption 4 eliminates the possibility of having two minimizers in (2). We consider the case where consequently, Lemma 1 also provides an analytical expression for the probability mass function (PMF) for user's choice of charging rate.

Lemma 1 (Corollary 1 of [5])**.** *Given Assumptions 1, 2, 3, and 4, consider the set of L functions of two independent RVs $\{g_\ell(x_j, \alpha_j)\}_{\ell=1}^L$ where each g_ℓ is as defined in* (1). *Then, for $k \in \{1, \ldots, L\}$,*

$$\mathbb{P}\big(S(x_j, \alpha_j) = k\big) = \mathbb{P}\left(\underline{\alpha}^k < \alpha_j < \bar{\alpha}^k\right)$$

where $\underline{\alpha}^1 = -\infty$ and $\bar{\alpha}^L = +\infty$ otherwise $\bar{\alpha}^k = \min_{k<i} \frac{\Delta_k^i V}{\Delta_i^k R}$, $\underline{\alpha}^k = \max_{i<k} \frac{\Delta_k^i V}{\Delta_i^k R}$. Furthermore, the charging rate r_j chosen by each user j is a discrete random variable each with PMF

$$p_r(r; p, a) = \begin{cases} \mathbb{P}\left(\underline{\alpha}^1 < \alpha_j < \bar{\alpha}^1\right) & \text{if } r = R^1\,, \\ \quad\vdots & \\ \mathbb{P}\left(\underline{\alpha}^L < \alpha_j < \bar{\alpha}^L\right) & \text{if } r = R^L\,. \end{cases} \tag{3}$$

Lemma 1 demonstrates that the probability of choosing a service level solely depends on the likelihood that a user j's impatience factor $\alpha_j \sim p_A(\alpha; p, a)$ falls within the interval $(\underline{\alpha}^k, \bar{\alpha}^k)$.

A charging facility with pricing functions of the form (1) experiences user arrivals with i.i.d impatience factors that are distributed with $p_A(\alpha; p, a)$. Any expected value that is dependent on $p_A(\alpha; p, a)$ is written as $\mathbb{E}[\,\cdot\,; p, a]$. Specifically, one can write the expected occupancy at a charging facility as $\mathbb{E}[\eta(t); p, a] = \lambda\,\mathbb{E}[x/r; p, a] = \lambda\,\mathbb{E}[x]\,\mathbb{E}[\frac{1}{r}; p, a]$.

Since we in this paper are interested in studying how the expected occupancy changes when users' impatience changes, we start by stating a general upper bound on the deviation of the expected occupancy.

Proposition 1. *Consider a charging facility operating under Assumptions 1, 2, 3, and 4. Assume that $\eta(t)$ represents the occupancy possibly with two different probability mass functions for the impatience factor, $p_A(\alpha; p, a)$ and $p_A(\alpha, \tilde{p}, \tilde{a})$, then*

$$|\mathbb{E}[\eta(t); p, a] - \mathbb{E}[\eta(t); \tilde{p}, \tilde{a}]| \leq \lambda\,\mathbb{E}[x]\left(\frac{1}{R^1} - \frac{1}{R^L}\right).$$

Moreover, there exists probability mass functions such that the bound is tight.

Proof: The choice of charging rate is independent of the user's demand, hence $\mathbb{E}[\eta(t); p, a] = \lambda\,\mathbb{E}[x]\,\mathbb{E}[\frac{1}{r}; p, a]$. Let $p_r(r; p, a)$ and $p_r(r; \tilde{p}, \tilde{a})$ denote the corresponding distributions for the charging rates, according to Lemma 1. We then obtain $|\mathbb{E}[\eta(t); p, a] - \mathbb{E}[\eta(t); \tilde{p}, \tilde{a}]| =$ $\lambda\,\mathbb{E}[x]\left|\sum_{\ell=1}^L \frac{p_r(R^\ell; p, a)}{R^\ell} - \sum_{\ell=1}^L \frac{p_r(R^\ell; \tilde{p}, \tilde{a})}{R^\ell}\right|$. From Proposition 1 it can be seen that it is possible to choose distributions of α such that $p_r(R^1; p, a) = 1$ or $p_r(R^L; p, a) = 1$, and $p_r(R^1; \tilde{p}, \tilde{a}) = 1 - p_r(R^1; p, a)$, $p_r(R^L; \tilde{p}, \tilde{a}) = 1 - p_r(R^L; p, a)$. □

Proposition 1 demonstrates that the difference between the true and mischaracterized expected occupancy is upper bounded. Assuming a correct characterization of λ and $\mathbb{E}[x]$, this worst-case upper bound is driven by the difference of the inverse of the fastest and slowest charging rate. In practice, a charging facility operator may not have a correct characterization of the arriving user's impatience factors but they can set the charging rates in such a way that the difference of the inverses of the slowest and fastest charging rate is small.

Given the PMF of choosing a particular charging rate from Lemma 1, we can derive an expression for how the expected occupancy $\mathbb{E}[\eta(t); p, a]$ varies with probability mass values. The following theorem formalizes the gradient of $\mathbb{E}[\eta(t); p, a]$ with respect to p.

Theorem 1. *Consider a charging facility operating under Assumptions 1, 2, 3, and 4 with L pricing functions of the form of* (1)*. Recall that $\mathbb{E}[\eta(t); p, a]$ is the expected occupancy where user's impatience factors are distributed with $p_A(\alpha; p, a)$. For any time t at steady state,*

$$\nabla_p\,\mathbb{E}[\eta(t); p, a] = \lambda\,\mathbb{E}[x]\begin{bmatrix} \sum_{\ell=1}^L \frac{\mathbb{1}_\ell(a_1)}{R^\ell} \\ \vdots \\ \sum_{\ell=1}^L \frac{\mathbb{1}_\ell(a_M)}{R^\ell} \end{bmatrix}, \tag{4}$$

where $\mathbb{1}_\ell(a_i) = 1$ *if* $\underline{\alpha}^\ell < a_i < \bar{\alpha}^\ell$ *and* $\mathbb{1}_\ell(a_i) = 0$ *otherwise.*

Proof: Given a charging facility operating under Assumptions 1, 2, 3, and 4 with L pricing functions of the form of (1). Recalling the PMF of α_j, we note that the probability of choosing a specified charge rate ℓ has an equivalence where $\mathbb{P}\left(\underline{\alpha}^\ell < \alpha_j < \bar{\alpha}^\ell\right) = \sum_{m=1}^{M} p_m \mathbb{1}_\ell(a_m)$. Recall that $\mathbb{E}[\eta(t); p, a] = \lambda\, \mathbb{E}[x]\, \mathbb{E}[\frac{1}{r}; p, a]$. Furthermore, one can expand on $\mathbb{E}[\frac{1}{r}; p, a]$ such that $\mathbb{E}\left[\frac{1}{r}; p, a\right] = \sum_{\ell=1}^{L} p_r(R^\ell; p, a) \frac{1}{R^\ell} = \sum_{\ell=1}^{L} \mathbb{P}\left(\underline{\alpha}^\ell < \alpha_j < \bar{\alpha}^\ell\right) \frac{1}{R^\ell} = \sum_{\ell=1}^{L} \left(\sum_{m=1}^{M} p_m \mathbb{1}_\ell(a_m)\right) \frac{1}{R^\ell}$. Given the prior substitutions, one can compute the gradient of $\mathbb{E}[\eta(t); p, a]$ with respect to p leading to (4). □

From (4) in Theorem 1 one can see that the gradient of $\mathbb{E}[\eta(t); p, a]$ is constant; hence, $\mathbb{E}[\eta(t); p, a]$ varies linearly with p. A direct corollary of Theorem 1 is that of the gradient of the expected occupancy when the probability mass values are mischaracterized. Specifically, consider the case where a charging facility is operating with the impatience category vector a but has mischaracterized the impatience probability mass vector to be $\tilde{p}$ such that $p_A(a_i; \tilde{p}, a) = \tilde{p}_i$. In reality, the true probability mass vector of the arriving users is p and hence the charging facility is operating under a mischaracterized impatience PMF. Observing that this result is analogous to Theorem 1 one concludes that when the probability mass vector is mischaracterized to be $\tilde{p}$ the error in the expectation $\mathbb{E}[\eta(t); p, a] - \mathbb{E}[\eta(t); \tilde{p}, a]$ varies linearly with $p - \tilde{p}$, i.e, $\nabla_{p-\tilde{p}}\left(\mathbb{E}[\eta(t); p, a] - \mathbb{E}[\eta(t); \tilde{p}, a]\right)$ is constant.

In addition to the variation of $\mathbb{E}[\eta(t); p, a]$ with respect to p, a charging facility is also interested in the variation of $\mathbb{E}[\eta(t); p, a]$ with respect to a. This is formalized in the following theorem.

Theorem 2. *Consider a charging facility operating under Assumptions 1, 2, 3, and 4 with L pricing functions of the form of* (1). *Recall that* $\mathbb{E}[\eta(t); p, a]$ *is the expected occupancy where user's impatience factors are distributed with* $p_A(\alpha; p, a)$. *For any time* t *at steady state,*

1) *For all* p, a *such that for every* a_i *there exists a* $k > 0$ *such that* $a_i \in (\underline{\alpha}^k, \bar{\alpha}^k)$, *it holds that*
$$\nabla_a \mathbb{E}[\eta(t); p, a] = 0\,.$$
2) *For all* a, *for all* i, *and for all* $k < L$,
$$\lim_{\tilde{a}_i \uparrow \bar{\alpha}^k} \mathbb{E}[\eta(t); p, \tilde{a}] - \lim_{\tilde{a}_i \downarrow \bar{\alpha}^k} \mathbb{E}[\eta(t); p, \tilde{a}] = \lim_{\epsilon \to 0^+} \lambda\, \mathbb{E}[x] \left(\sum_{\ell=1}^{L} \frac{p_i}{R^\ell} \left(\mathbb{1}_\ell(\bar{\alpha}^k - \epsilon) - \mathbb{1}_\ell(\bar{\alpha}^k + \epsilon) \right) \right), \tag{5}$$
where $\tilde{a}_j = a_j$ *for some* j, *and* $\mathbb{1}_\ell(a_m) = 1$ *if* $\underline{\alpha}^\ell < a_m < \bar{\alpha}^\ell$ *and* $\mathbb{1}_\ell(a_m) = 0$ *otherwise.*

Proof: To prove the first part of Theorem 2, recall $\mathbb{E}[\eta(t); p, a] = \lambda\, \mathbb{E}[x]\, \mathbb{E}[\frac{1}{r}; p, a]$. Furthermore, $\mathbb{E}\left[\frac{1}{r}; p, a\right] = \sum_{\ell=1}^{L} \left(\sum_{m=1}^{M} p_m \mathbb{1}_\ell(a_m)\right) \frac{1}{R^\ell}$ where $\mathbb{1}_\ell(a_m) = 1$ if $\underline{\alpha}^\ell < a_m < \bar{\alpha}^\ell$ and $\mathbb{1}_\ell(a_m) = 0$ otherwise for any $m \in \{1, \ldots, M\}$. Since we have p, a such that $a_i \in \left(\underline{\alpha}^k, \bar{\alpha}^k\right)$ for some k for all i then the quantity $\mathbb{E}\left[\frac{1}{r}; p, a\right]$ is constant with respect to a. This is because $a_i \in (\underline{\alpha}^k, \bar{\alpha}^k)$ and since the probability mass remains in the interval there is no change to $\mathbb{E}[\eta(t); p, a]$. Hence, $\nabla_a \mathbb{E}[\eta(t); p, a] = 0$.

To prove the second part of Theorem 2, we analyze the difference $\mathbb{E}[\eta(t); p, a^-] - \mathbb{E}[\eta(t); p, a^+]$ where $\mathbb{E}[\eta(t); p, a^-] = \lim_{\tilde{a}_i \uparrow \bar{\alpha}^k} \mathbb{E}[\eta(t); p, \tilde{a}]$ and $\mathbb{E}[\eta(t); p, a^+] = \lim_{\tilde{a}_i \downarrow \bar{\alpha}^k} \mathbb{E}[\eta(t); p, \tilde{a}]$ and $\tilde{a}_j = a_j$ for some j. From before, realize that $\mathbb{E}[\eta(t); p, a^-] = \lambda\, \mathbb{E}[x]\, \mathbb{E}[\frac{1}{r}; p, a^-]$ and $\mathbb{E}[\eta(t); p, a^+] = \lambda\, \mathbb{E}[x]\, \mathbb{E}[\frac{1}{r}; p, a^+]$. Substituting the summation form of $\mathbb{E}[\frac{1}{r}; p, a^-]$ and $\mathbb{E}[\frac{1}{r}; p, a^+]$ leads to (5). □

Theorem 2 first states that the expected occupancy does not vary when a impatience value a_i is changed within a given $(\bar{\alpha}^k, \underline{\alpha}^k)$ interval. The second part of the theorem states what happens if the change of a_i crosses the boundary of an interval $(\bar{\alpha}^k, \underline{\alpha}^k)$, i.e., a change in the expected occupancy.

IV. Numerical Study

In this section, we present a numerical study that illustrates practical occurrences of Proposition 1, and Theorem 1 and 2.[1] Consider a charging facility offering $L = 3$ service levels that is capacity-constrained on the expected occupancy and is deciding between offering two sets of prices and charging rates to meet this constraint. Specifically, this charging facility can either operate under the pricing scheme A where the prices are $V_A^1 = 0.15$, $V_A^2 = 0.25$ and $V_A^3 = 0.32$ in \$/kWh. and the charging rates are $R_A^1 = 15$, $R_A^2 = 30$ and $R_A^3 = 35$ in kW. Alternatively, the charging facility can use pricing scheme B where $V_B^1 = 0.05$, $V_B^2 = 0.25$ and $V_B^3 = 0.33$ in \$/kWh. and $R_B^1 = 15$, $R_B^2 = 30$ and $R_B^3 = 35$ in kW. In this particular case, a charging facility has a fixed set of charging rates to offer at each service level but is deciding what prices to charge for energy at each service level. At this charging facility user arrivals are a Poisson process with $\lambda = 30$ EVs/hr. Furthermore, user's energy demands (in kWh.) are distributed with $x_j \sim f_X(x_j)$ where $f_X(x_j) = \mathcal{U}(5, 100)$. Note that $\mathcal{U}(\cdot)$ denotes a uniform distribution.

Lastly, the charging facility operator estimates user's impatience factor to be a PMF that is defined to be $p_A(\alpha; \tilde{p}, \tilde{a})$. We specifically consider the case when there are $M = 4$ impatience profiles that describe the user population and assume that the charging facility is correctly estimating that $M = 4$ profiles also exist such that $\tilde{a} = [\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{a}_4]^\top$.

Given the aforementioned prices and charging rates for pricing schemes A and B we consider the following two potential cases a charging facility may experience: first, a case when the true distribution of the impatience of users is in fact a discrete random variable with $p = \tilde{p}$ and $a \neq \tilde{a}$ and secondly when the true distribution is not a discrete random variable but in fact a bounded multi-modal normal distribution. In considering these cases, we are able

[1] The Python code for this case study is available at `https://github.com/gtfactslab/cdc2021_evcharging_sensitivity`

to illustrate which of the two pricing schemes is robust to these types of errors by quantifying the error in the expected occupancy from the PMF estimate the facility has made and the two scenarios for the true distributions of the user impatience.

First consider the case when a charging facility has estimated the user impatience PMF to have parameters $\tilde{p} = [0.25, 0.25, 0.25, 0.25]^\top$ and $\tilde{a} = [2, 10, 20, 25]^\top$ but in reality the users impatience distributions are $p = \tilde{p}$ but $a = [2, 5.5, 20, 25]^\top$. The discrepancy between a and $\tilde{a}$ will result in differing values for $\mathbb{E}[\eta(t); p, a]$ and $\mathbb{E}[\eta(t); \tilde{p}, \tilde{a}]$, respectively.

We present Fig. 1 which illustrates $p_A(\alpha_j; \tilde{p}, \tilde{a})$ and $p_A(\alpha_j; p, a)$ in the top and bottom plots, respectively, with the respective values $(\underline{\alpha}^k, \bar{\alpha}^k)$ for both pricing schemes are displayed. Recall from Lemma 1 that the probability mass of $p_A(\alpha_j; \tilde{p}, \tilde{a})$ or $p_A(\alpha_j; p, a)$ in the non-empty intervals $(\underline{\alpha}^k, \bar{\alpha}^k)$ determines the probability of users choosing service level k. As a result a discrepancy between a and $\tilde{a}$ can lead to different expected occupancy between the estimates and what occurs in practice. While problematic, a charging facility can mitigate such a discrepancy by choosing a pricing scheme that is resilient to such mischaracterizations.

Given the pricing schemes A and B with $\tilde{a}$ we illustrate the intervals $(\underline{\alpha}^k, \bar{\alpha}^k)$ in the plots of Fig. 1. Recall $S(x_j, \alpha_j)$ from (2), then the choices made by user j in pricing schemes A and B are $S_A(x_j, \alpha_j)$ and $S_B(x_j, \alpha_j)$, respectively. Then, a charging facility with estimate $\tilde{a}$ concludes that $\mathbb{P}(S_A(x_j, \alpha_j) = 1) = \mathbb{P}(S_A(x_j, \alpha_j) = 2) = 0.25$ and $\mathbb{P}(S_A(x_j, \alpha_j) = 3) = 0.50$, and $\mathbb{P}(S_B(x_j, \alpha_j) = 1) = \mathbb{P}(S_B(x_j, \alpha_j) = 2) = 0.25$ and $\mathbb{P}(S_B(x_j, \alpha_j) = 3) = 0.50$.

In reality, the true impatience values a demonstrated in the bottom plot of Fig. 1 show that $\mathbb{P}(S_A(x_j, \alpha_j) = 1) = \mathbb{P}(S_A(x_j, \alpha_j) = 2) = 0.25$ and $\mathbb{P}(S_A(x_j, \alpha_j) = 3) = 0.50$ and that $\mathbb{P}(S_B(x_j, \alpha_j) = 1) = 0.50$, $\mathbb{P}(S_B(x_j, \alpha_j) = 2) = 0$ and $\mathbb{P}(S_B(x_j, \alpha_j) = 3) = 0.50$. In practice, for pricing scheme B this states that even though a charging facility is offering 3 service levels, only 2 of them will be chosen by users. Numerically, this leads to an over 20% error between $\mathbb{E}[\eta(t); p, a]$ and $\mathbb{E}[\eta(t); \tilde{p}, \tilde{a}]$ when a charging facility utilizes pricing scheme B due to the charging facility erroneously underestimating the expected occupancy. However, in this case, no error arises when using pricing scheme A.

While this is illustrative, this represents the considerations a charging facility must make when setting energy prices or charging rates to make their pricing scheme resilient to mischaracterizations of users' impatience. In analyzing the variability of the expected occupancy when utilizing $\tilde{a}$ or a one can see an illustration of Statement 1 of Theorem 2. Specifically, if $a_1 > \bar{\alpha}^1$ for pricing scheme B then Statement 1 of Theorem 2 would predict that the expectation would have remained the same. However, since $a_1 < \bar{\alpha}^1$ for pricing scheme B the difference in the expectation is as predicted in Statement 2 of Theorem 2. We illustrate the variation of the expected occupancy with varying $\tilde{a}$ for both pricing schemes in Fig. 2 when both the estimated and true impatience are discrete random variables. Lastly, consider the

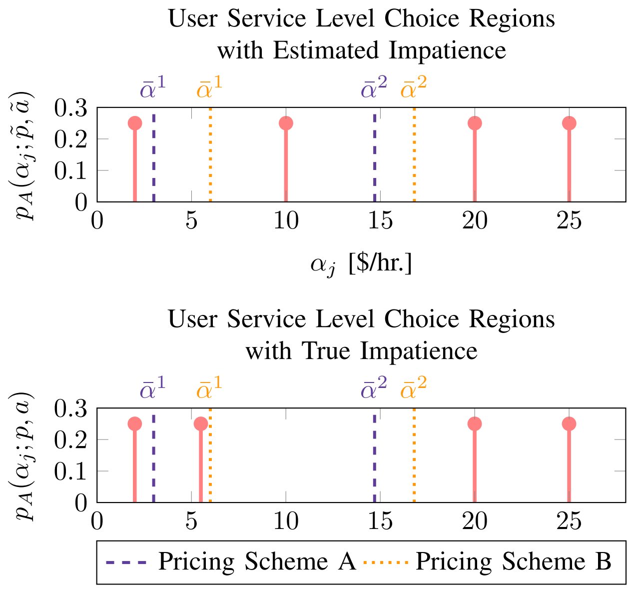

Fig. 1. A charging facility decides between pricing scheme A or B when it has estimated the user impatience as in this figure. In reality, we suppose the user impatience at \$10 per hr. is really \$5.5 per hr.. The discrepancy between the estimated impatience values $\tilde{a}$ and the true impatience values a leads to the charging facility having an incorrect estimate of the expected occupancy. As described in Section IV, pricing scheme A leads to the same expected occupancy under both the true and estimated impatience, while pricing scheme B leads to expected occupancy that is over 20% larger for the true impatience distribution compared to the estimated impatience distribution.

hypothetical scenario where $\tilde{a} = a$ but $\tilde{p} \neq p$, then one can see from analyzing Fig. 1 that $\mathbb{E}[\eta(t); p, a]$ will vary linearly with p.

Secondly, we consider the case when a charging facility has estimated the impatience to be a discrete random variable as before but the true user impatience distribution is a truncated multi-modal normal distribution with probability density function (PDF) $f_A(\alpha_j)$. Given the distribution illustrated in Fig. 3 we have that $\mathbb{P}(S_A(x_j, \alpha_j) = 1) = 0.221$, $\mathbb{P}(S_A(x_j, \alpha_j) = 2) = 0.281$ $\mathbb{P}(S_A(x_j, \alpha_j) = 3) = 0.498$ and that $\mathbb{P}(S_B(x_j, \alpha_j) = 1) = 0.431$, $\mathbb{P}(S_B(x_j, \alpha_j) = 2) = 0.070$ and $\mathbb{P}(S_B(x_j, \alpha_j) = 3) = 0.499$. This leads to an over 15% error in the expected occupancy between the estimated and true user impatience when a charging facility utilizes pricing scheme B. Furthermore, pricing scheme A now has an approximately 3% error from the true value where the charging facility has overestimated the expected occupancy. The increase in error in pricing scheme A in this scenario is due to the portion from the first mode of $f_A(\alpha_j)$ that is in $(\underline{\alpha}^2, \bar{\alpha}^2)$, i.e., $(\bar{\alpha}^1, \bar{\alpha}^2)$, of pricing scheme A and the portion of the second mode of $f_A(\alpha_j)$ from the first mode that is in $(\underline{\alpha}^2, \bar{\alpha}^2)$, i.e., $(\bar{\alpha}^1, \bar{\alpha}^2)$, of pricing scheme B.

We present the numerical values from this numerical study in Table I. In both scenarios where the true impatience distributions are a PMF and a PDF, respectively, we see that pricing scheme A is more resilient to mischaracterizations of $p_A(\alpha_j; \tilde{p}, \tilde{a})$ (or $f_A(\alpha_j)$) than pricing scheme B. In fact, pricing scheme B leads the charging facility operator to esti-

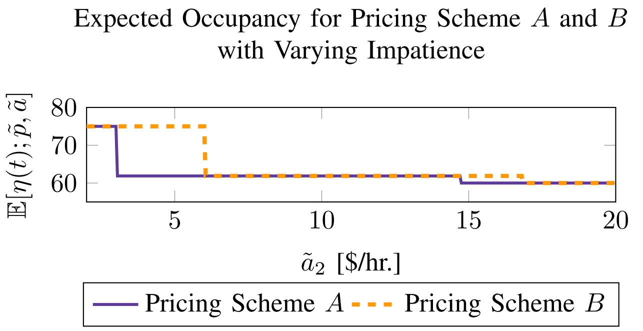

Fig. 2. An EV charging facility where users arrive according to a Poisson process with random demand and impatience decides between a Pricing Scheme A and B. At arrival, users choose a charging rate from a collection of service levels that minimizes the total cost to themselves that includes their impatience. A charging facility has estimated the impatience PMF for the arriving users and utilizes this estimate to compute the expected occupancy for both pricing schemes. In this plot, we illustrate the variation in the expected occupancy when $\tilde{a} = [2, \tilde{a}_2, 20, 25]^\top$. As a charging facility's estimate of $\tilde{a}_2$ varies so does the expected occupancy under each pricing scheme.

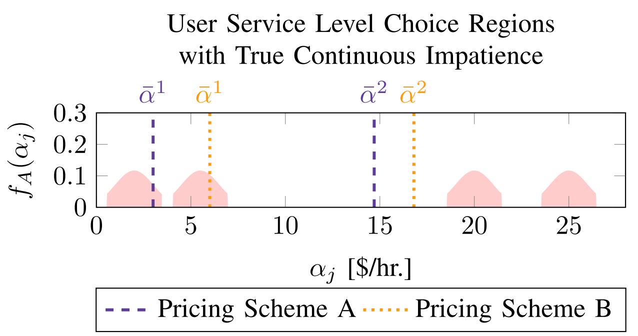

Fig. 3. A charging facility decides between implementing pricing scheme A or B when it has estimated the user impatience as in the top plot of Fig. 1. In reality, we suppose the user impatience is a truncated multi-modal normal distribution as is shown in this plot. Similar to the case in Fig. 1, Pricing Scheme A leads to true expected occupancy that is within 3% of the estimated expected occupancy, while Pricing Scheme B leads to true expected occupancy that is over 15% higher than estimated.

mate a lower occupancy when in reality the value is higher. This potentially overburdens the facility's space resources. In practice, an operator gains insightful information from the operation of a charging facility on users' impatience that will guide them in choosing mischaracterization-resilient prices and charging rates.

V. Conclusion

In this paper, we studied the problem of a charging facility operating with a defined service level model where users arrive randomly with a collection of random parameters. We specifically focus on the case where a charging facility is primarily interested in characterizing the expected occupancy at the charging facility. To compute the expected occupancy, a charging facility utilizes its knowledge on the distributions of the user arrivals and the respective parameters (energy demand and impatience factor). While useful, these computations are vulnerable to incorrect assessments by the charging facilities of the distribution of user parameters. Specifically, within the model, computing the expected occupancy is highly dependent on having the correct knowledge of the distribution of user's impatience. As a result, we study the variability in the expected occupancy when the distribution and values of the impatience factor are mischaracterized. Furthermore, we also compute a worst-case error bound for the expected occupancy when the impatience factor is mischaracterized. We study the analytical results via a numerical study that illustrates how a charging facility operator can intelligently set prices and charging rates to minimize the effects of mischaracterized user's impatience.

TABLE I
Expected Occupancy for Different Distributions and Pricing

Probability Distribution	Expected Occupancy Pricing Scheme A	Expected Occupancy Pricing Scheme B
Estimated Impatience	61.875	61.875
True Discrete Impatience	61.875	75.0
True Multi-Modal Impat.	60.38	71.40

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