Probabilistic Revenue Analysis for Electric Vehicle Charging

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Abstract— **Electric Vehicle (EV) charging facilities operate under specific pricing models to mitigate the often random demands of arriving users while balancing their self-interested financial goals. In this paper, we study charging facilities operating with a discrete service level model. Here, users arrive randomly with a collection of random demands. In particular, an arriving user selects the service level, i.e., energy price and charging rate, that minimizes the total cost of receiving service. Upon selection, a portion of the service level cost faced by users, a function of the offered prices and rates, becomes revenue to the charging facility. To that end, we consider the case when a charging facility has a collection of charging rates to offer such that the respective prices maximize the expected revenue. First, we present an optimization program that yields the service level prices that maximize the expected revenue at a charging facility with consideration for the charging facility's operational costs. Then, we derive a high-confidence bound on the total revenue expected at a charging facility operating under the service level model. Lastly, we illustrate the application of the results via a numerical study.**

I. INTRODUCTION

Governments are pushing for, and consumers are active participants in, the continued adoption of electric vehicles (EVs). The rate of this adoption poses both operational and financial challenges for EV charging facilities. From an operational perspective, EV charging facilities can employ a variety of operational models, e.g., bidding pricing models [1], [2], scheduling models [3], [4], etc., to mitigate the effects of the increased demand for their services. On the financial side, charging facilities must carefully weigh the effects of their pricing methodologies on users and their own financial goals and requirements.

For example, two recent studies point to the need for further investment in the standardization of the availability of on-site commercial charging, estimating that \$110-180 billion needs to be invested in private and commercial charging infrastructure to satisfy global charging demand [5], [6]. Thus, the increased demands on charging infrastructure from the increased adoption of EVs call for further capital expenditures from both governments and charging facility operators. While some of the required investment for EV charging facilities will come from government infrastructure investments, the remainder of the investment will be shouldered by EV charging facility operators and their stakeholders. This fact drives the need to analyze the financial trade-offs of operating an EV charging facility given a particular operational model. Hence, given the estimated magnitude of the required future investment, we are motivated to study the revenue side of charging facilities.

In general, the EV charging problem has been addressed from four overarching perspectives: pricing-aware EV charging models, routing and schedule-based EV charging, EV charging station design, and data-driven EV charging [7], [8]. In this work, we focus on pricing-aware EV charging models; specifically, we are interested in the application of specific pricing models which provide charging facility revenue-side insights upon their deployment.

Current EV charging literature focused on pricing takes disparate approaches; for example, the papers [9], [10], study the problem of optimal pricing at EV charging facilities where the charging facility and the utility provider are separate, independent entities and users solve a path-dependent decision problem which minimizes the cost to themselves. These works are extended in [7] where the charging facility operators set prices based on congestion pricing techniques. In the papers [11], [12] it is demonstrated that it is possible to create a pricing mechanism that can actively accept or reject users based on system-wide parameters such as demand while handling the inherent randomness of the EV reservation system. The paper [13] studies the EV charging revenue problem by adapting capacity control mechanisms from asset revenue management in order to allocate charging capacity. Lastly, the paper [14] studies the revenue maximization problem at plug-in hybrid charging stations by studying the equilibria of customer subscription dynamics.

In the present work, we study the revenue problem of EV charging facilities operating under a service level model introduced in [15]. Specifically, this model is called the defined service level (DSL) model where users arrive randomly at the charging facility. Upon arrival, users have a collection of random parameters which quantify their energy demands and their value of time (opportunity cost). Using these parameters, users choose a service level, i.e., energy price and charging rate, from the set of offered service choices which minimizes the total cost to themselves. A portion of the total cost faced by users is received as revenue by the charging facility. Previous works such as [15], [16] have focused on the operation of charging facilities with considerations for finite resources. In this work, we focus on

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setting charging rate prices that yield revenue maximization and on providing high-confidence bounds on the achieved revenue levels at the charging facilities.

The contributions of this paper are three-fold: first, we present a revenue side model for the charging facility which augments the user-side model presented in [15]. Second, we utilize the charging facility revenue model to construct an optimization program that sets the service level prices while maximizing the charging facility's expected revenue. Third, we derive a concentration bound that provides a confidence interval on the likelihood of exceeding specified revenue levels given knowledge of the random parameters of the arriving users and the revenue-maximizing service level prices. Throughout this paper, we consider charging facilities whose lowest service level operating costs are subsidized by a government authority. The presence of a subsidy is motivated by current federal and state laws which subsidize the development and operation of EV charging facilities [17]. To the best of our knowledge, this is the first service level model which considers the effects of pricing subsidies on setting the charging facility revenue-maximizing prices and the respective revenue projections.

This paper is organized as follows: Section II formulates the EV charging problem, Section III presents the revenue-maximizing optimization program and the revenue confidence bound, and Section IV and V present a numerical study of the main results and the work's conclusions, respectively.

A. Notation

For an indexed set of variables $\{x^k\}$, we let $\Delta^i_j x$ denote the difference between the variable with index i and j, i.e., $\Delta^i_j x = x^i - x^j$. When considering a collection of independent and identically distributed (i.i.d) random variables indexed by subscripts, we use non-subscript variables when referring to properties that hold for any of the i.i.d random variables. For example, $\mathbb{E}[x]$ is the expectation of each i.i.d random variable x_j.

II. Problem Formulation

In this section, we present a pricing model for EV charging that was initially introduced in [15], [16]. We consider a *defined service level (DSL) model* where users directly choose from a discrete set of charging rates and prices upon arrival at the charging facility; a user pays a higher price for a faster charge rate. Furthermore, we supplement this model by introducing a charging facility-side revenue model that accounts for operational costs.

A rational user chooses a charge rate depending on the amount of charge required for their EV, the prices and rates set by the charging facility, and their impatience factor, i.e., their value of time. In this paper, we assume that users generally prefer to minimize the cost to themselves and depart the facility immediately upon receiving a full charge.

At this facility, a user j arrives at some time τ_j (in hr.) with charging demand x_j (in kWh), and an impatience factor α_j (in \$/hr.). Throughout the paper, we make the following assumption about the aforementioned variables.

TABLE I
User Parameter Definitions

Var.	Parameter	Unit	Range
j	user index	-	-
τ_j	arrival time	hr.	-
x_j	user demand	kWh	$[x_{\min}, x_{\max}]$
α_j	impatience factor	\$/hr.	$\{a_1, \dots, a_M\}$
r_j	charging rate	kW	$(0, R^{\max}]$

TABLE II
Parameter Definitions for the Charging Facility

Var.	Parameter	Unit	Range
ℓ	service level	-	$\{1, \dots, L\}$
V^ℓ	price per unit of energy	\$/kWh	-
R^ℓ	charging rate	kW	$(0, R^{\max}]$
W^ℓ	operating cost	\$/kWh	-

Assumption 1 (Users): User arrivals at the charging facility are a Poisson process with parameter λ (in EVs/hr.). Individual charging demand x_j, and the impatience factor α_j for each user j are random variables which are independent and identically distributed (i.i.d). In particular, x_j and α_j are continuous random variables with support $[x_{\min}, x_{\max}]$ for some $0 < x_{\min} < x_{\max}$ and $[\alpha_{\min}, \alpha_{\max}]$ for some $0 < \alpha_{\min} < \alpha_{\max}$, respectively.

The user parameters, their respective units, and upper and lower bounds are summarized in Table I. The charging facility offers L service levels. Each service level $\ell \in \{1, \dots, L\}$ corresponds to a distinct charging rate $R^\ell > 0$ (in kW) and price $V^\ell > 0$ (in \$/kWh) that is the cost per unit energy for the service level. Thus, user j with energy demand x_j pays $x_j V^\ell$ (in \$) to receive a full charge over the time horizon x_j / R^ℓ (in hr.) when choosing service level ℓ.

The parameters related to the charging facility under a discrete pricing model are listed in Table II. To distinguish the parameters related to the charging facility from those related to the users, the charging facility parameters are upper case and indexed by a superscript, while the parameters for the users are lower case and indexed by a subscript j.

Assumption 2 (Model Charging Rates): Among L service levels offered by the charging facility, a higher charging rate is more costly, i.e., if $R^i > R^k$ then $V^i > V^k$. Moreover, charging rates and prices are distinct so that $R^i \neq R^k$ for all $i \neq k$. Lastly, and without loss of generality, the charging facility's pricing functions are enumerated such that $V^1 < V^2 < \ldots < V^L$ and therefore $R^1 < R^2 < \ldots < R^L$.

A user can therefore pay less by choosing a slower charge rate but must balance this with their impatience. In particular, the total cost faced by a user arriving at the charging facility with impatience factor α_j, charging demand x_j, and who chooses service level ℓ, is

$$g_\ell(x_j, \alpha_j) = x_j V^\ell + \alpha_j \frac{x_j}{R^\ell}\,. \tag{1}$$

In (1), the first term of the sum is the energy cost to the user and the second term is the cost associated with how much a user values their time.

Individual users choose a service level at a charging facility which minimizes their total cost of charging factoring in their impatience. To that end, let $S(x_j, \alpha_j) : [x_{\min}, x_{\max}] \times \{a_1, \dots, a_M\} \to \{1, \dots, L\}$ be defined by

$$S(x_j, \alpha_j) = \underset{\ell \in \{1,\dots,L\}}{\arg\min}\; g_\ell(x_j, \alpha_j)\,. \tag{2}$$

Then, a rational user j chooses service level $S(x_j, \alpha_j)$ in order to minimize their total cost as formalized in the later stated assumption.

For notational convenience, we also define the values r_j to be the charging rate and cost per unit of energy chosen by user j after solving (2), i.e., $r_j = R^{S(x_j,\alpha_j)}$, as indicated in Table I. An arriving user chooses a service level according to (2) and the probability of each choice is formulated in the following Lemma.

Lemma 1 (Corollary 2 from [15]): Under Assumptions 1, 2, and 3, consider the set of L functions of two independent RVs $\{g_\ell(x_j, \alpha_j)\}_{\ell=1}^{L}$ where each g_ℓ is of the DSL model as defined in (1). Then, for $k \in \{1, \dots, L\}$,

$$\mathbb{P}\big(S(x_j, \alpha_j) = k\big) = \mathbb{P}\left(\underset{\sim}{\alpha}^k < \alpha_j < \bar{\alpha}^k\right)$$

where

$$\bar{\alpha}^k = \min\left(\alpha_{\max},\ \min_{k<i} \frac{\Delta_k^i V}{\Delta_i^k \bar{R}}\right), \tag{3}$$

$$\underset{\sim}{\alpha}^k = \max\left(\alpha_{\min},\ \max_{i<k} \frac{\Delta_k^i V}{\Delta_i^k \bar{R}}\right) \tag{4}$$

where $\Delta_i^k V = V^k - V^i$ and $\Delta_i^k \bar{R} = \bar{R}^k - \bar{R}^i = 1/R^i - 1/R^k$ for all i, k. Furthermore, the charging rates r_j chosen by each user j is a collection of independent and identically distributed discrete random variables each with PMF

$$p_r(r) = \begin{cases} \mathbb{P}\left(\underset{\sim}{\alpha}^1 < \alpha_j < \bar{\alpha}^1\right) & \text{if } r = R^1\,, \\ \quad\vdots & \\ \mathbb{P}\left(\underset{\sim}{\alpha}^L < \alpha_j < \bar{\alpha}^L\right) & \text{if } r = R^L\,. \end{cases} \tag{5}$$

Observe that the user charging times x_j/r_j, being uniquely determined by x_j and α_j, constitute a collection of independent and identically distributed random variables. Furthermore, this means the time a user spends at the charging location is x_j/r_j where this is the time for a user to receive a full charge based on their chosen service level.

Assumption 3 (Users are Rational): Each user chooses a charging rate according to (2) and leaves the charging facility once they have satisfied their charging demand. Thus, user j occupies a charger at the facility during the time interval $[\tau_j, \tau_j + x_j/r_j]$.

Define η_{total} be the total number of users which pass through a charging facility during an operation duration.

Charging facilities are interested in understanding their financial gain from the provided energy levels. Recall that we are considering charging facilities whose lowest service level is subsidized by a government entity. This is formalized in the following assumption.

Assumption 4 (Operational Subsidy): The charging facility operation is subsidized such that the fixed cost to the charging facility for operating the lowest service level is zero and the price of the lowest service level is fixed where the price value for the first service level V^1 is not at the discretion of the charging facility. However, the charging facility may set the service level prices $V^2, \dots, V^L$ at their discretion.

In practice, the presence of the subsidy outlined in Assumption 4 means that an external entity, e.g., a government entity, fixes the lowest price offered at a charging facility in exchange for covering the first service levels operational costs. The existence of this subsidy creates an interesting dynamic for the charging facility as it seeks to maximize its charging revenue when setting the prices $V^2, \dots, V^L$.

When an arriving user makes a choice of service level at the charging facility part of the total cost to themselves is seen as revenue to the charging facility. Let the revenue per user be the random variable z such that the revenue received by a charging facility is

$$z(x_j, \alpha_j) = \begin{cases} x_j(V^1 - W^1) & \text{if } S(x_j, \alpha_j) = 1 \\ x_j(V^2 - W^2) & \text{if } S(x_j, \alpha_j) = 2 \\ \quad\vdots & \\ x_j(V^L - W^L) & \text{if } S(x_j, \alpha_j) = L \end{cases}$$

where W^ℓ (in \$/kWh) is a fixed cost for operating the service level ℓ at the charging facility. Furthermore, define $f_Z(z)$ is the distribution of revenue per service level. Note that in $z(x_j, \alpha_j)$ that $W^1 = 0$ due to a subsidy to the lowest service level detailed in Assumption 4. While a charging facility is motivated to maximize their revenue, the existence of a subsidy and anchoring of the lowest service level pricing creates a scenario of competing priorities when charging facilities set their service level prices.

Note that since x_j is a bounded random variable and V^ℓ and W^ℓ for $\ell \in \{1, \dots, L\}$ are bounded values, there exists a $Z^{\max}$ such that $z(x_j, \alpha_j) < Z^{\max}$ for all $S(x_j, \alpha_j) = i$ where $i \in \{1, \dots, L\}$.

Given that $z(x_j, \alpha_j)$ is dependent on $S(x_j, \alpha_j)$ one can break down the expected revenue conditioned on $S(x_j, \alpha_j)$. Then, from the law of total expectation we know that

$$\mathbb{E}[z] = \sum_{i=0}^{L} \mathbb{E}[z \mid S(x_j, \alpha_j) = i]\,\mathbb{P}(S(x_j, \alpha_j) = i).$$

Lastly, define

$$Y = \sum_{i=1}^{\eta_{\text{total}}} z_i$$

to be the total revenue received by the charging facility. Note that Y is a compound Poisson process [18].

III. Main Results

In this section, we present two results that enable a charging facility to understand the financial dynamics of their operation and the subsequent effects on users from such pricing policies. We utilize the service model from Section II where a charging facility receives revenue from arriving users who choose a service level, i.e, price and charging rate, according to (2). Here, we present an optimization program that maximizes the facility's revenue. Furthermore, given a set of revenue-maximizing service level prices, a charging facility is interested in the likelihood of achieving specific revenue levels. Hence, we derive a high-confidence concentration bound which gives a confidence interval on the likelihood of a charging facility achieving a particular revenue level.

At the moment a user chooses a service level dependent on their random parameters, the charging facility receives some revenue from the user's choice. A charging facility sets prices to maximize this revenue; however, since the revenue received from each user is a random variable we utilize the expected revenue per user $\mathbb{E}[z]$ as the objective for the optimization program and maximize its value. Then, the optimization program to set the service level prices at a charging facility is as follows.

Program 1 (Optimize Service Level Prices):

$$\begin{aligned} \max_{V^m, m\in\{2,\dots,L\}} \quad & \mathbb{E}[z] \\ \text{subject to} \quad & \\ V^i < & V^k \text{ for } i < m \text{ where } i,k \in \{1,\dots,L\} \end{aligned}$$

Program 1 formalizes a charging facility setting the service level prices while maximizing the expected revenue. In addition, Program 1 maintains the price ordering of service levels from Assumption 2. Moreover, we study this problem in the context with the presence of a subsidy for service level $\ell = 1$ as formalized in Assumption 4 such that V^1 is fixed and $W^1 = 0$.

Program 1 yields the per user revenue-maximizing service level prices; however, in practice, a charging facility is also interested in the total revenue received during a time period of operation such that it enables facility operators to better forecast their investment capabilities. Specifically, since users arrive randomly with random demands the charging facility must make probabilistic estimates on what revenue levels it will attain.

Recall from Section II that Y is the total revenue received from η_{total} arrivals at the charging facility over a time period of length T. Note that Y is a random sum whose distribution is not easily obtained. Hence, quantifying the confidence with which a charging facility will attain specific revenue levels is challenging. As a result, we resort to utilizing concentration inequalities to derive a confidence bound on achieving specific revenue levels. We formalize the confidence bound on achieving specific revenue levels given a specific set of user parameters in the following theorem.

Theorem 1: Consider a charging facility operating under the DSL model with Assumptions 1, 2, and 3. Given any $\mathcal{S} \geq 0$ total revenue, the following statement holds:

With confidence $1 - \phi(\mathcal{S})$, where

$$\phi(\mathcal{S}) = \begin{cases} \min\left\{1, \sum_{m=\lceil \frac{\mathcal{S}}{\mathbb{E}[z]} \rceil}^{\infty} \exp\left(\frac{-(m\,\mathbb{E}[z]-\mathcal{S})^2}{2\left(m\,\mathbb{E}[z^2]+\frac{Z^{\max}(m\,\mathbb{E}[z]-\mathcal{S})}{3}\right)}\right) \right. \\ \left. \times \mathbb{P}(\eta = m)\right\}, \quad \text{if } \mathcal{S} < \mathbb{E}[\eta_{\text{total}}]\,\mathbb{E}[z] \\ 1, \qquad \text{otherwise,} \end{cases}$$

the total revenue for all users will exceed $\mathcal{S}$, i.e., $\mathbb{P}(Y > \mathcal{S}) \geq 1 - \phi(\mathcal{S})$.

Proof: Introduce $\nu = \eta_{total}\,\mathbb{E}[z] - \mathcal{S}$. Hence, $\mathcal{S} = \eta_{total}\,\mathbb{E}[z] - \nu$. From total probability we know that,

$$\mathbb{P}(Y \leq \mathcal{S}) = \sum_{m=0}^{\infty} \mathbb{P}\big(Y \leq \mathcal{S} \mid \eta_{\text{total}} = m\big)\mathbb{P}\big(\eta_{\text{total}} = m\big)\,.$$

Using Bernstein's inequality from Fact 1 for the lower tail of a distribution we can bound the conditional probability as follows.

$$\mathbb{P}\big(Y \leq \mathcal{S} \mid \eta_{\text{total}}\big) \leq \exp\left(\frac{-(\eta_{total}\,\mathbb{E}[z]-\mathcal{S})^2}{2\left(\eta_{total}\,\mathbb{E}[z^2]+\frac{Z^{\max}(\eta_{\text{total}}\,\mathbb{E}[z]-\mathcal{S})}{3}\right)}\right)$$

Then, utilizing this fact within the summation which leads to the following.

$$\mathbb{P}(Y \leq \eta_{\text{total}}\,\mathbb{E}[z] - \nu \mid \eta_{\text{total}}) \leq \sum_{m=0}^{\infty} \exp\left(\frac{-(m\,\mathbb{E}[z]-\mathcal{S})^2}{2\left(m\,\mathbb{E}[z^2]+\frac{Z^{\max}(m\,\mathbb{E}[z]-\mathcal{S})}{3}\right)}\right)\mathbb{P}\big(\eta_{\text{total}} = m\big)\,.$$

where $m\,\mathbb{E}[z] - \mathcal{S} > 0$. This implies that $m > \mathcal{S}\,/\,\mathbb{E}[z]$. Using this fact leads to

$$\mathbb{P}(Y \leq \mathcal{S}) \leq \sum_{m=\lceil \frac{\mathcal{S}}{\mathbb{E}[z]} \rceil}^{\infty} \exp\left(\frac{-(m\,\mathbb{E}[z]-\mathcal{S})^2}{2\left(m\,\mathbb{E}[z^2]+\frac{Z^{\max}(m\,\mathbb{E}[z]-\mathcal{S})}{3}\right)}\right) \times \mathbb{P}\big(\eta_{\text{total}} = m\big) = \phi^{\dagger}(\mathcal{S})\,.$$

Given the use of Bernstein's inequality we know that the upper bound on $\mathbb{P}(Y \leq \mathcal{S})$ will be less than 1 on some domain $\mathcal{S} \in (-\infty, \Gamma_{\mathcal{S}})$. To find the exact interval for when this occurs requires finding a specific value of $\mathcal{S}$; however, we know that $\Gamma_{\mathcal{S}}$ must be less than or equal to $\mathbb{E}[\eta_{\text{total}}]\,\mathbb{E}[z]$ as a result of using Bernstein's inequality on Y. Hence, we can define

$$\phi(\mathcal{S}) = \begin{cases} \min\left\{1, \phi^{\dagger}(\mathcal{S})\right\} & \text{if } \mathcal{S} < \mathbb{E}[\eta_{\text{total}}]\,\mathbb{E}[z] \\ 1 & \text{else.} \end{cases}$$

Recalling that $\mathbb{P}(Y > \mathcal{S}) = 1 - \mathbb{P}(Y \leq \mathcal{S}) > 1 - \phi(\mathcal{S})$ completes the proof. ■

IV. Numerical Study

In this paper, we study how a charging facility can set prices that maximize revenue and how the revenue-maximizing prices can be utilized to provide high-confidence revenue projections at a charging facility whose user parameter and arrival distributions are known. In particular, for a given charging facility we solve Program 1 to set prices and illustrate Theorem 1 to provide high-confidence revenue projections.

Consider an EV charging facility offering $L = 3$ service levels where the charging facility operator knows that they will offer the charging rates $R^1 = 5$ kW., $R^2 = 45$ kW, and $R^3 = 115$ kW. Each of the offered charging rates carries a cost of operation that is converted to a monetary rate per kWh. of charge; specifically, $W_1 = 0.0$ \$/kWh., $W_2 = 0.40$ \$/kWh., and $W_3 = 0.60$ \$/kWh. Note that since $W_1 = 0$ \$/kWh. due to the subsidy, i.e., no cost is incurred for offering the lowest service level, as detailed in Assumption 4.

Furthermore, a charging facility has estimated the users to arriving with Poisson process parameter $\lambda = 20$ EVs/hr. The arriving users have a continuous impatience distribution that is a truncated Rayleigh distribution $f_A(\alpha_j) \sim \mathcal{D}(5, 20)$ and the energy demand is a uniform distribution $f_X(x_j) \sim \mathcal{U}(10, 100)$.

We first consider the scenario where a charging facility operator is setting the prices according to Program 1. Here, a charging facility is setting the prices V^1, V^2, V^3 such that the expected revenue $\mathbb{E}[z]$ is maximized. Note that we study this problem in the context where subsidies are present such that $V_1 = 1.0$ \$/kW. is fixed. Given the aforementioned parameters on the user and the charging facility the maximizing service level prices are $V_2 = 2.426$ \$/hr. and $V_3 = 2.659$ \$/hr. and $P(S(x_j, \alpha_j) = 1) = 0.36$, $P(S(x_j, \alpha_j) = 2) = 0.45$, $P(S(x_j, \alpha_j) = 3) = 0.19$. Hence, a charging facility is able to maximize its revenue and maintain usage of all the offered service levels given the profile of arriving users.

After setting the prices at the charging facility using Program 1 a charging facility operator is also interested in computing a confidence level on the likelihood the charging facility will exceed a specific revenue level, i.e., $\mathbb{P}(Y > \mathcal{S})$. However, since finding a closed-form expression for the distribution of Y is difficult we utilize a lower bound on this probability such that $\mathbb{P}(Y > \mathcal{S}) \geq 1 - \phi(\mathcal{S})$.

We consider the scenario where the charging facility is operating for a time period of $T = 10$ hrs. Using V^ℓ and W^ℓ, we compute the confidence bound on achieving increasing revenue levels in Fig. 1. In Fig. 1 we see that the theoretical bound from Theorem 1 lower bounds the Monte Carlo value which is taken to be $\mathbb{P}(Y > S)$. Specifically, note that the larger the total revenue value the looser the confidence bound becomes. In other words, as the total revenue value increases a charging facility becomes less and less confident that such a value will be exceeded. This follows the intuition that achieving value further from the mean gets less and less likely. While the bound is conservative, it does allow charging facilities to appropriately forecast the revenue they will receive from the arriving user revenues when the distribution of Y is not known.

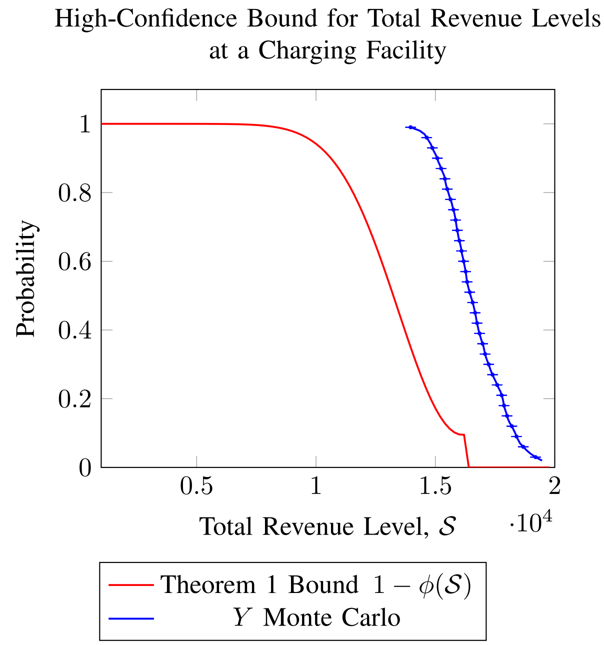

Fig. 1. A charging facility operating under the defined service level (DSL) model offers $L = 3$ service levels with charging rates $R^1 = 5$ kW., $R^2 = 45$ kW., and $R^3 = 115$ kW. Given these charging rates, a charging facility maximizes their revenue per user charge by solving Program 1. Ultimately, the charging facility operates with prices $V^1 = 1.0$, $V_2 = 2.426$ and $V_3 = 2.659$ \$/kWh. Moreover, a charging facility also needs to have a high-confidence of achieving specific revenue levels. We present the confidence bounds along with a Monte Carlo simulation of a charging facility operating under these parameters.

V. Conclusion

In this paper, we study the revenue problem for a charging facility operating under a defined service level (DSL) model where users arrive at a charging facility randomly with random parameters. A charging facility is interested in two aspects of its operation: 1) a charging facility must set the prices of the collection of charging rates offered to users while maximizing their revenue and 2) given a collection of prices and charging rates a charging facility makes high confidence projections on achieving specific revenue levels. Upon arrival, users choose the service level, i.e., energy price and charging rate, which minimizes the total cost to themselves. Each user choice presents a revenue stream to the charging facility, and hence, a charging facility operator must carefully select the service level prices. Upon selecting the prices, a charging facility can utilize a derived concentration bound along with knowledge of the statistics of the user parameters to compute confidence levels on achieving specific revenue thresholds. We illustrate both contributions via a numerical study.

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VI. Appendix

Fact 1 (Bernstein's Inequality, [19]): Given n independent, zero-mean random variables X_i such that, for some $b > 0$, $\nu > 0$, $0 \leq X_i \leq b$ almost surely for all $1 \leq i \leq n$. Then, it holds that

$$\mathbb{P}\left(\sum_{i=1}^{n}(X_i - \mathbb{E}[X_i]) \leq -\nu\right) \leq \exp\left(\frac{-\nu^2}{2\left(\sum_{i=1}^{n}\mathbb{E}[X_i^2] + \frac{b\nu}{3}\right)}\right). \quad (6)$$